

Absorption of electromagnetic radiation and non-equilibrium superconductivity

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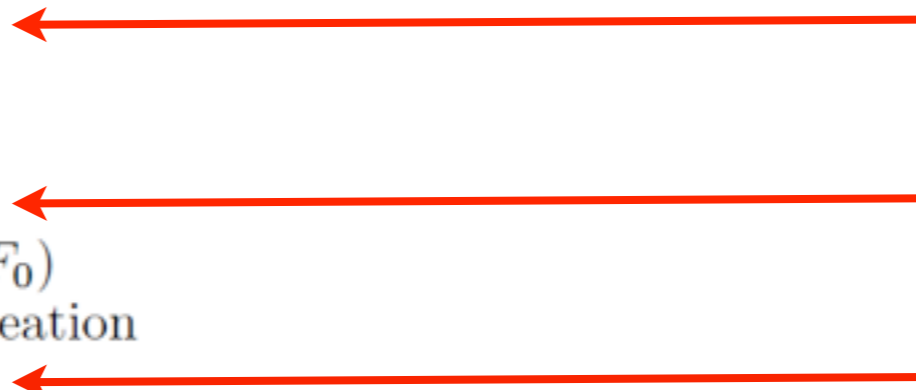
Photons 'heat' electrons and/or break pairs

- Superconducting tunnel-junctions (STJ's)
- Normal metal absorber and a TES as thermometer
- Microwave kinetic inductance detectors

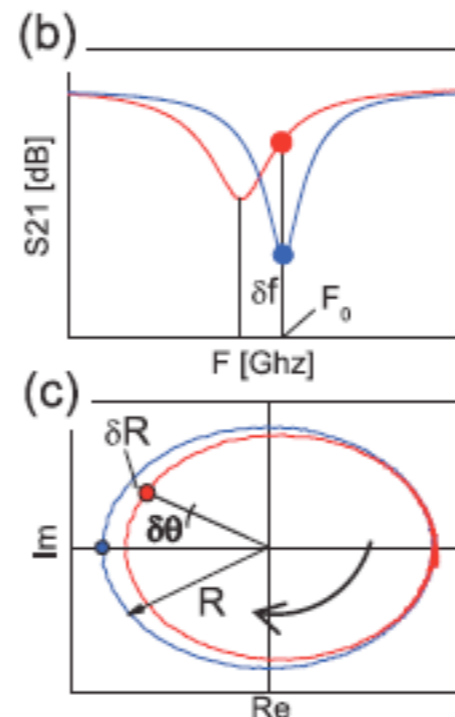
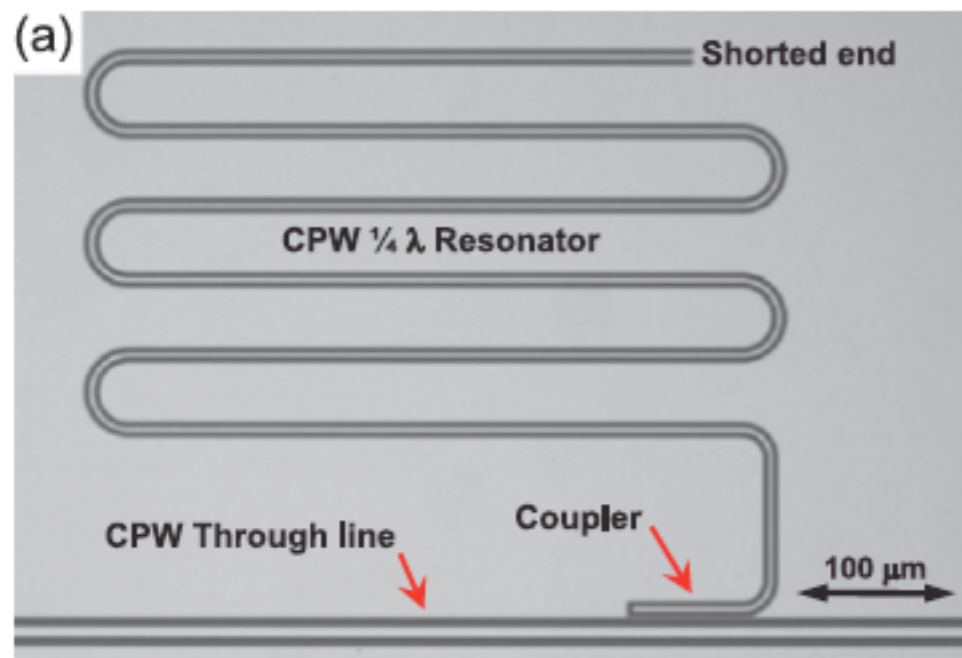
Observable S_{21}

$$NEP^2(\omega) = S_x \cdot \left[\frac{\eta\tau}{\Delta} \cdot \frac{\delta x}{\delta N_{qp}} \right]^{-2} (1 + \omega^2\tau^2)(1 + \omega^2\tau_{res}^2)$$

- ω is the frequency of the output spectrum
- x is Θ (for phase read-out) and R (for phase read-out)
- N_{qp} is the density of quasiparticles
- S_Θ is the phase noise
- S_R is the amplitude noise
- τ is the quasiparticle lifetime
- τ_{res} is the resonator ringtime ($Q/\pi F_0$)
- η is the efficiency of quasiparticle creation
- Δ is the superconducting gap



?



Aluminium
Titaniumnitride

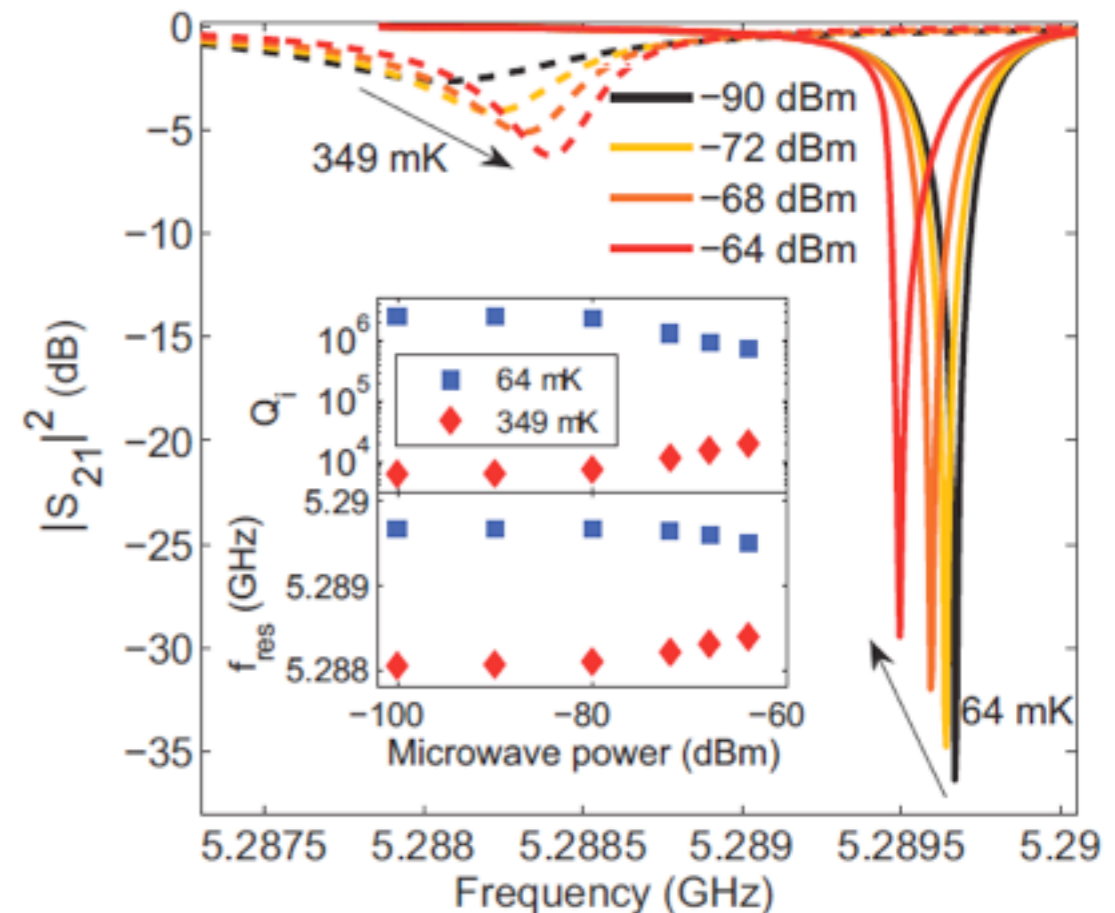
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Two frequencies: GHz and THz

- Optical photons (to be detected): create quasi-particles (break pairs)
- Microwave signal (used for read-out): determine non-equilibrium superconducting state

Non-equilibrium superconductivity

- Excess quasi-particles (δN_{qp})
- $f(E)$ electron-distribution
- $n(\Omega)$ phonon-distribution



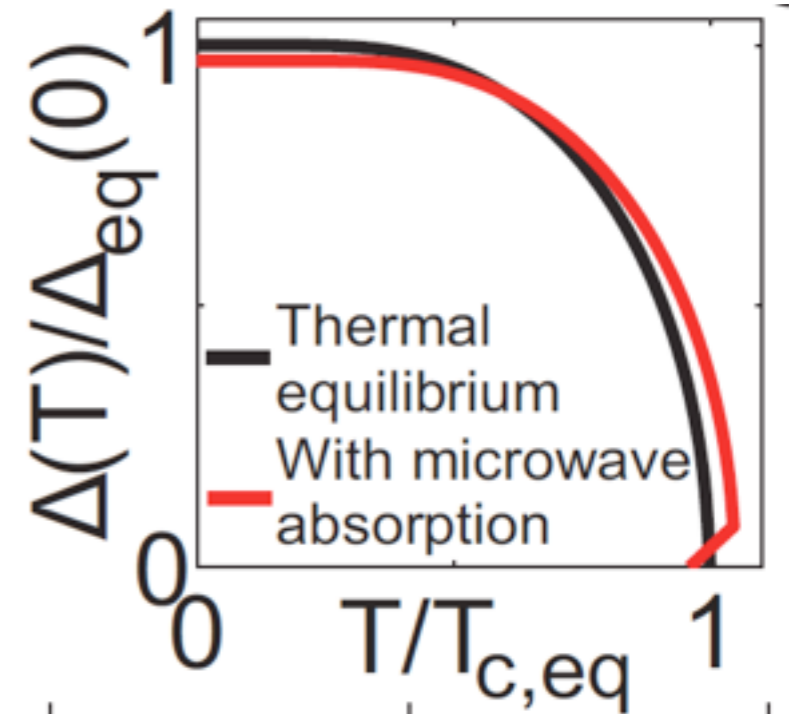
Counterintuitive results:

De Visser, Goldie, Diener, Withington, Baselmans, Klapwijk, arXiv 1306.4992 and LTD-15

Control parameter: $f(E)$

$$n_{\text{qp}} = 4N_0 \int_{\Delta}^{\infty} dE \frac{E}{\sqrt{E^2 - \Delta^2}} f(E)$$

$$\frac{1}{N_0 V_{BCS}} = \int_{\Delta}^{\infty} \frac{1 - 2f(E)}{\sqrt{E^2 - \Delta^2}} dE$$



Normally: $f(E) = \frac{1}{1 + e^{E/k_B T}}$

$$\frac{\Delta(T)}{\Delta(0)} \approx \left[1 - \frac{T}{T_c}\right]^{\frac{1}{2}}$$

All BCS expressions in which the temperature appears is through $f(E)$. No change in: V_{BCS}

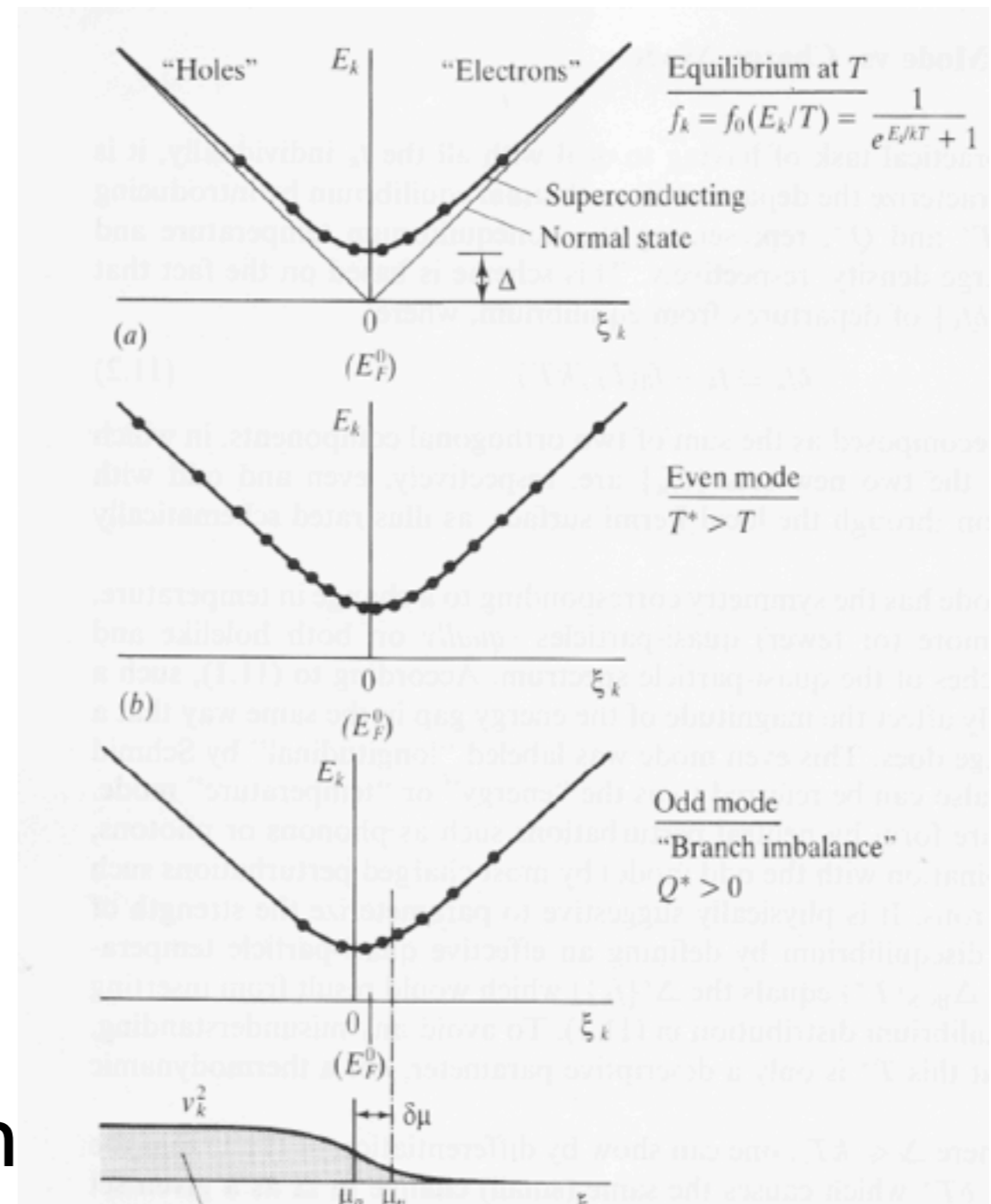
Even and odd $f(E)$

- Transversal and longitudinal nonequilibrium
- Charge and energy-mode nonequilibrium
- Asymmetric and symmetric nonequilibrium

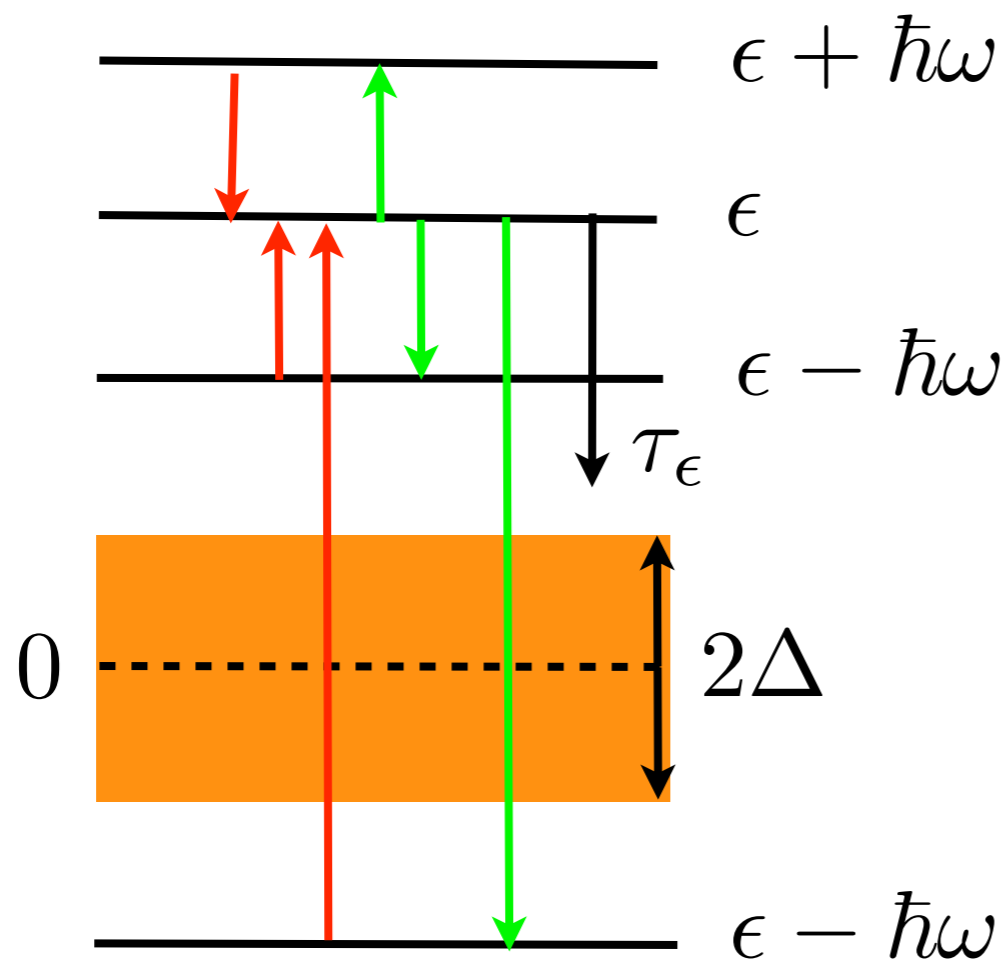
$$q_k = \frac{\xi_k}{E_k} = \frac{[E_k^2 - \Delta_k^2]^{1/2}}{E_k}$$

$$Q^* = \sum_k q_k \delta f_k$$

cf. NS current conversion



Microwave absorption + relaxation

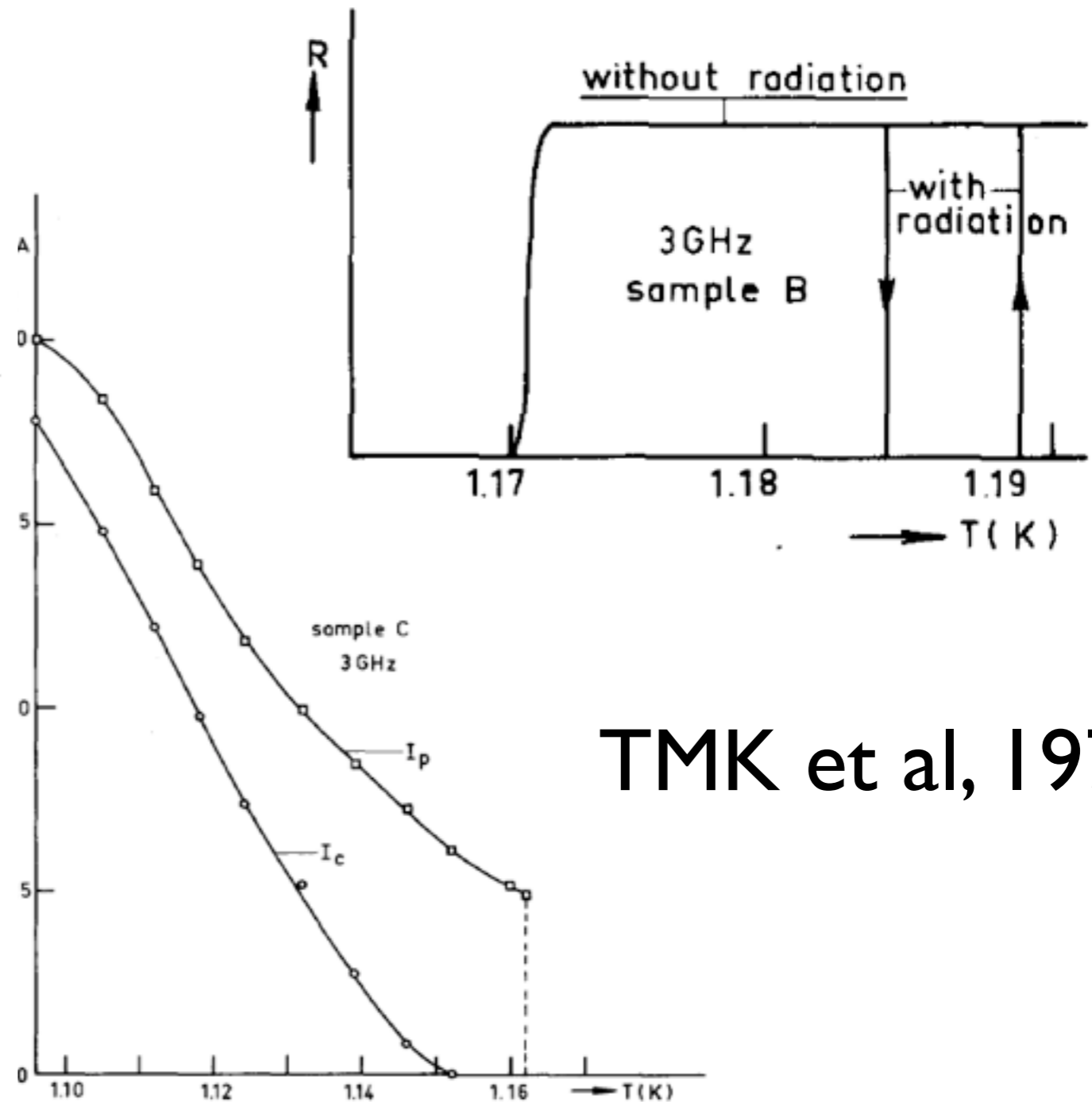
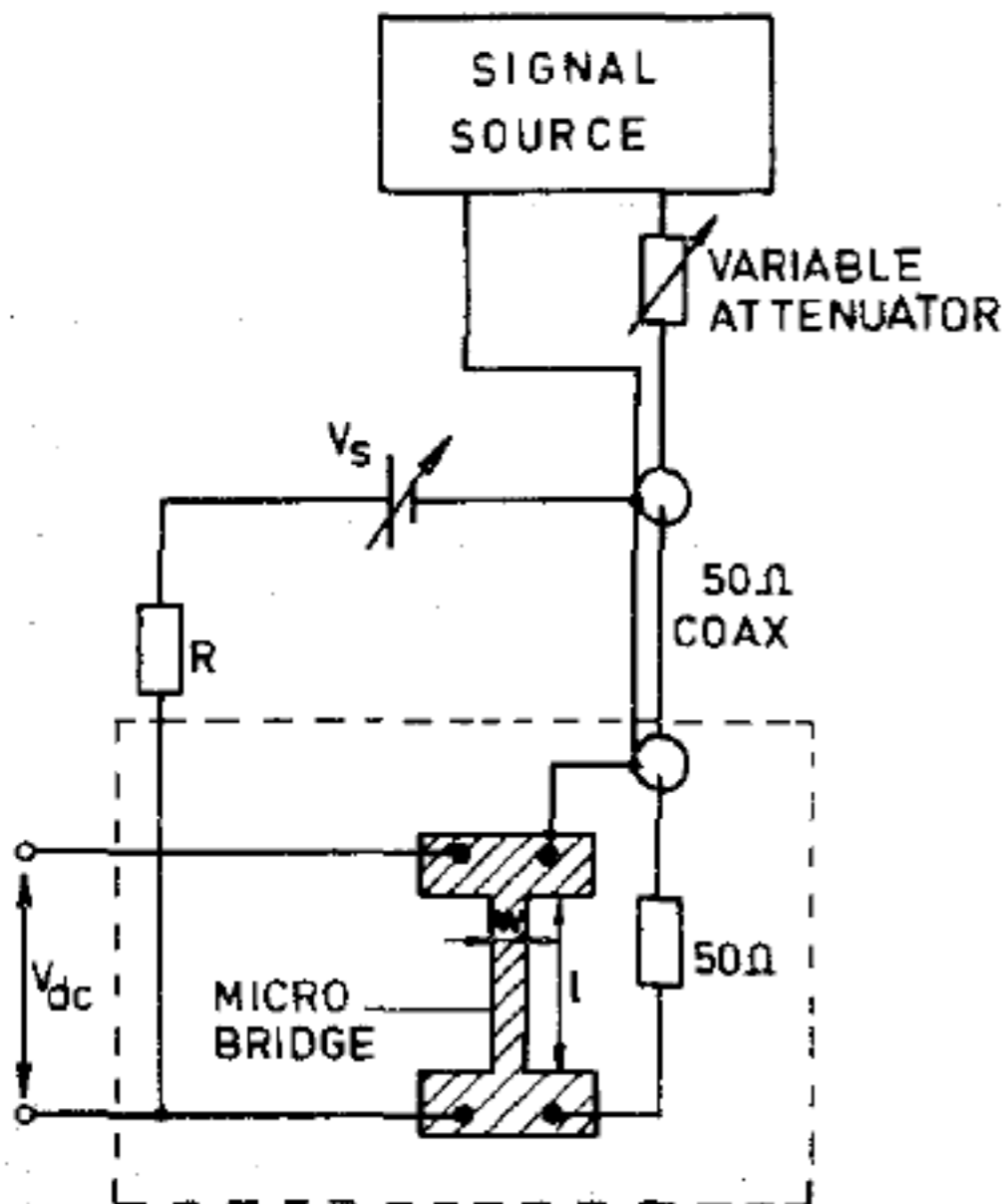


$$n_1(\varepsilon) = \frac{\alpha}{\gamma} \frac{\hbar\omega}{4kT_c} \left[\frac{\varepsilon(\varepsilon - \hbar\omega) + \Delta^2}{\varepsilon[(\varepsilon - \hbar\omega)^2 - \Delta^2]^{1/2}} \theta(\varepsilon - \Delta - \hbar\omega) - \frac{\varepsilon(\varepsilon + \hbar\omega) + \Delta^2}{\varepsilon[(\varepsilon + \hbar\omega)^2 - \Delta^2]^{1/2}} \theta(\varepsilon - \Delta) - \frac{\varepsilon(\varepsilon - \hbar\omega) + \Delta^2}{\varepsilon[(\varepsilon - \hbar\omega)^2 - \Delta^2]^{1/2}} \theta(\varepsilon - \Delta) \theta(\hbar\omega - \Delta - \varepsilon) \right]$$

$$\gamma = \frac{\hbar}{\tau_\varepsilon}$$

$$\alpha = \frac{De^2}{\hbar c^2} A_\omega^2$$

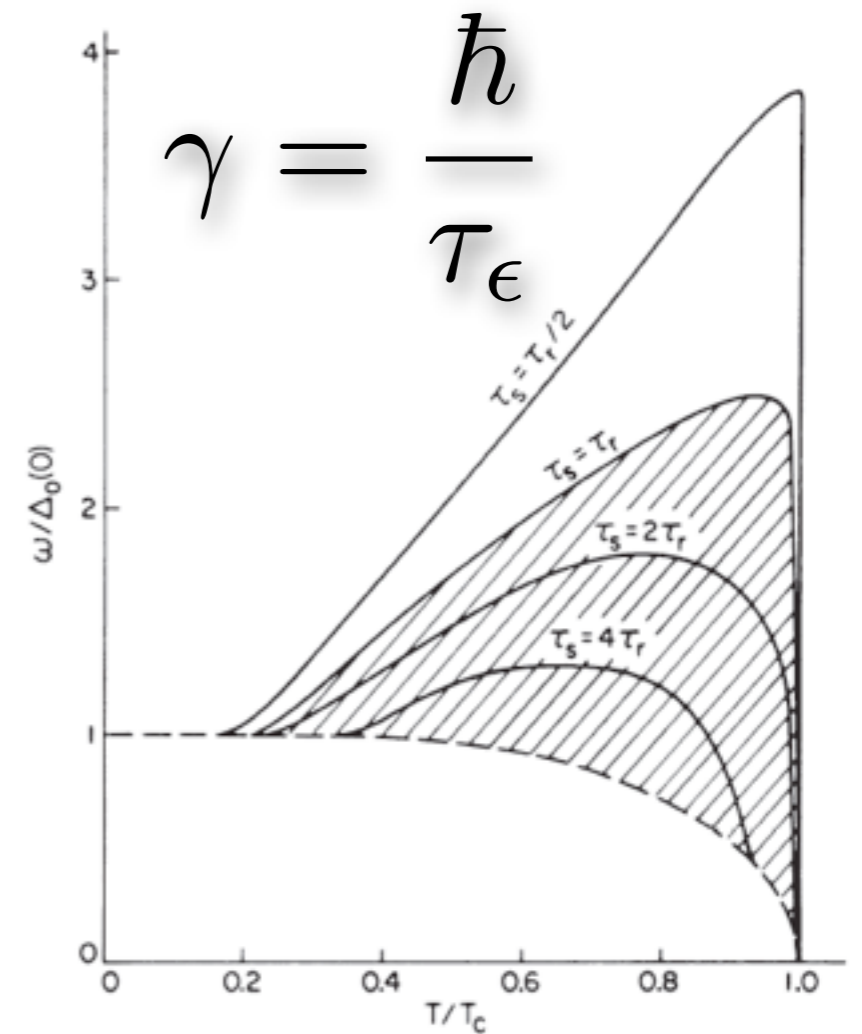
Microwave-enhanced superconductivity



TMK et al, 1976

Energy-dependent relaxation processes

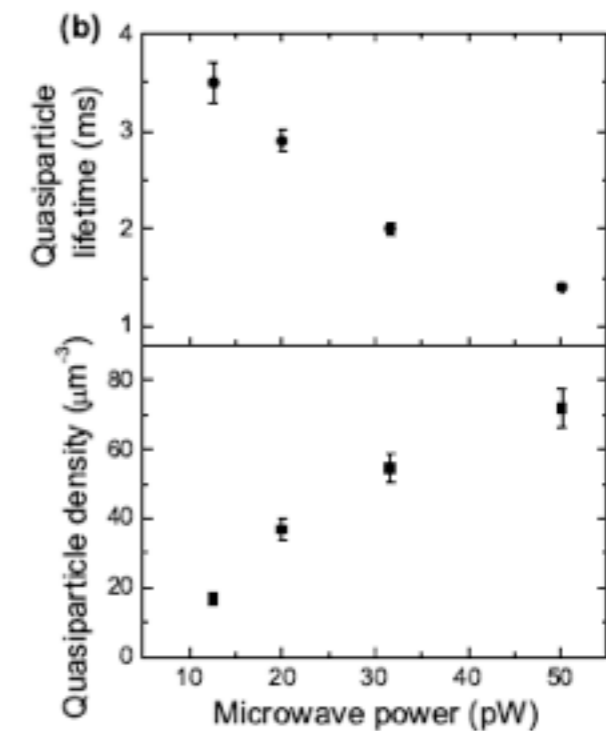
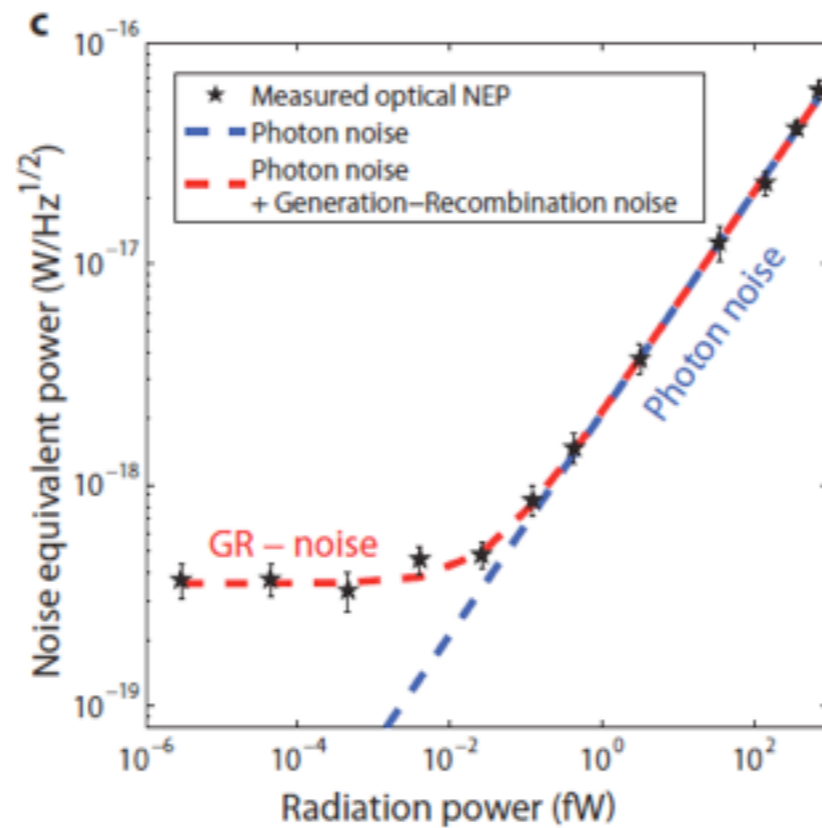
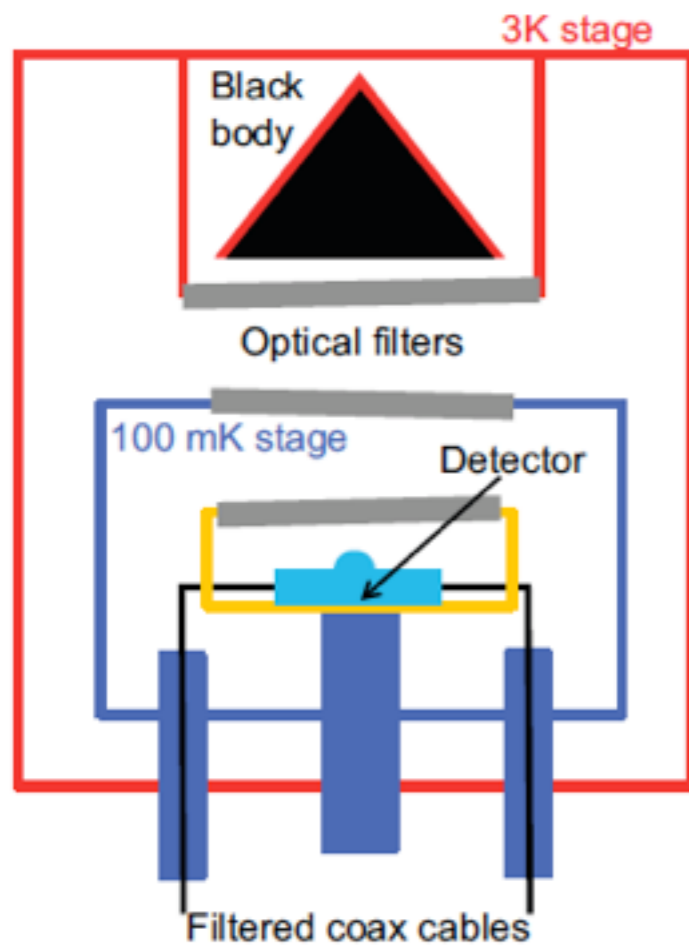
- Kaplan et al (1976)
- Chang-Scalapino (1978)
- Enhanced recombination rates at higher energies



Aluminium

- Weak electron-phonon interaction
- Therefore high conductivity at room temperature
- Low T_c
- Slow energy-relaxation times; slow recombination times

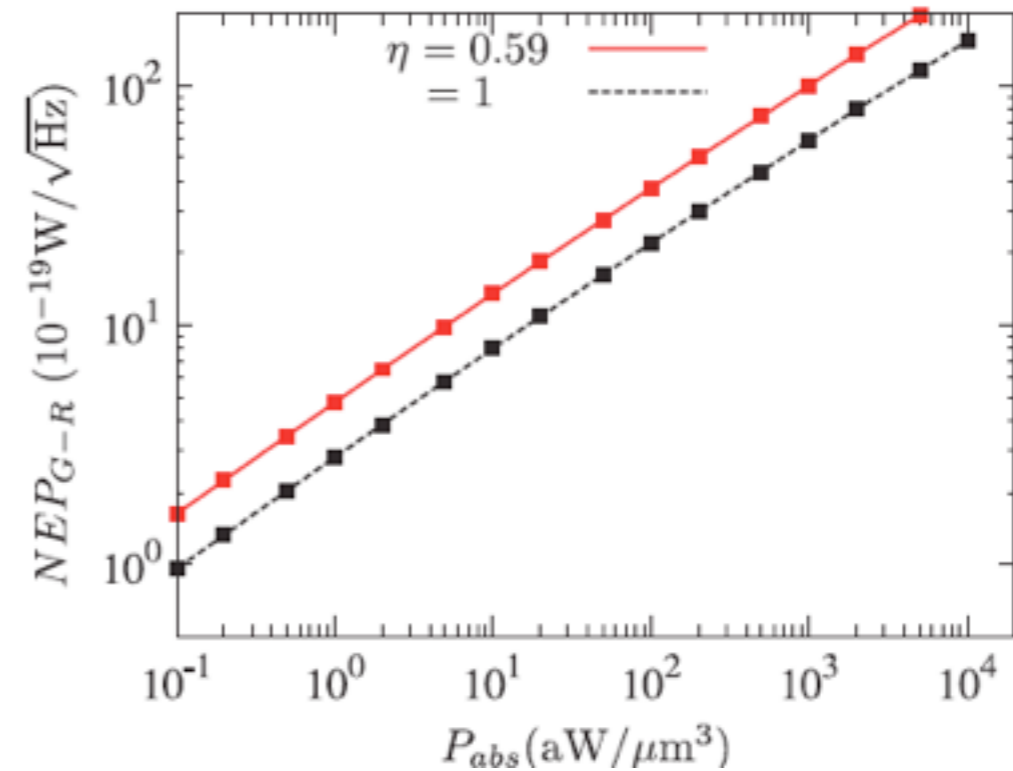
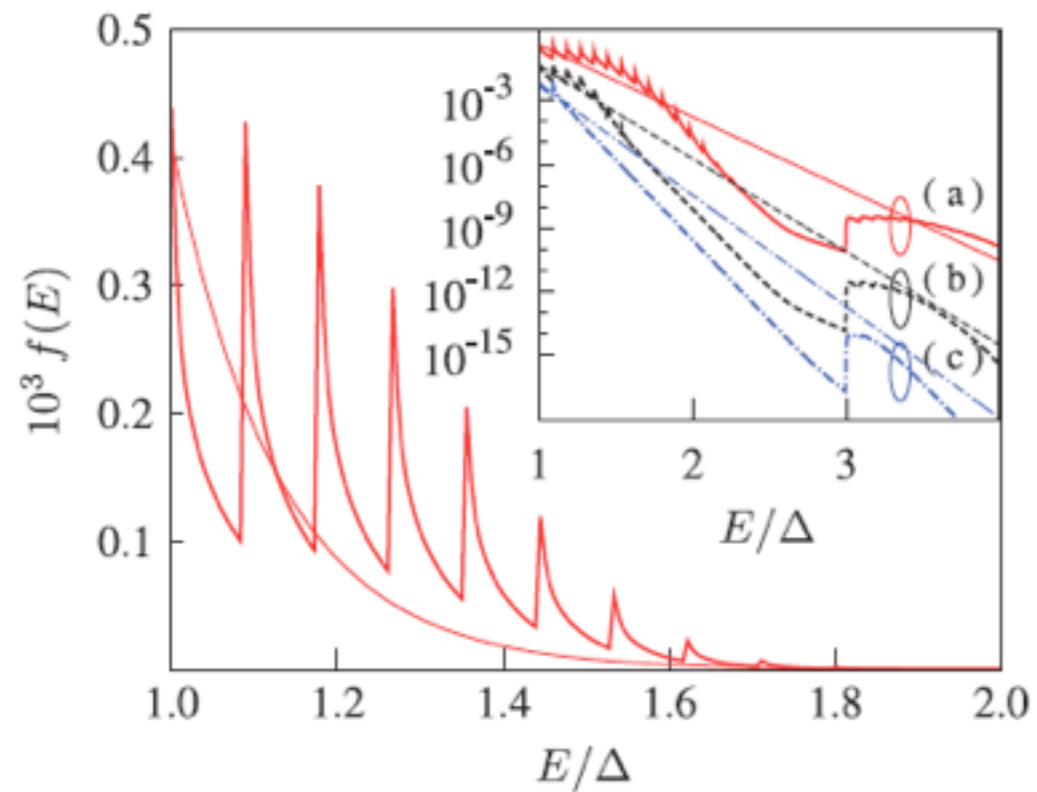
NEP limited by background quasiparticle density



De Visser et al, APL **100**, 162601 (2012)

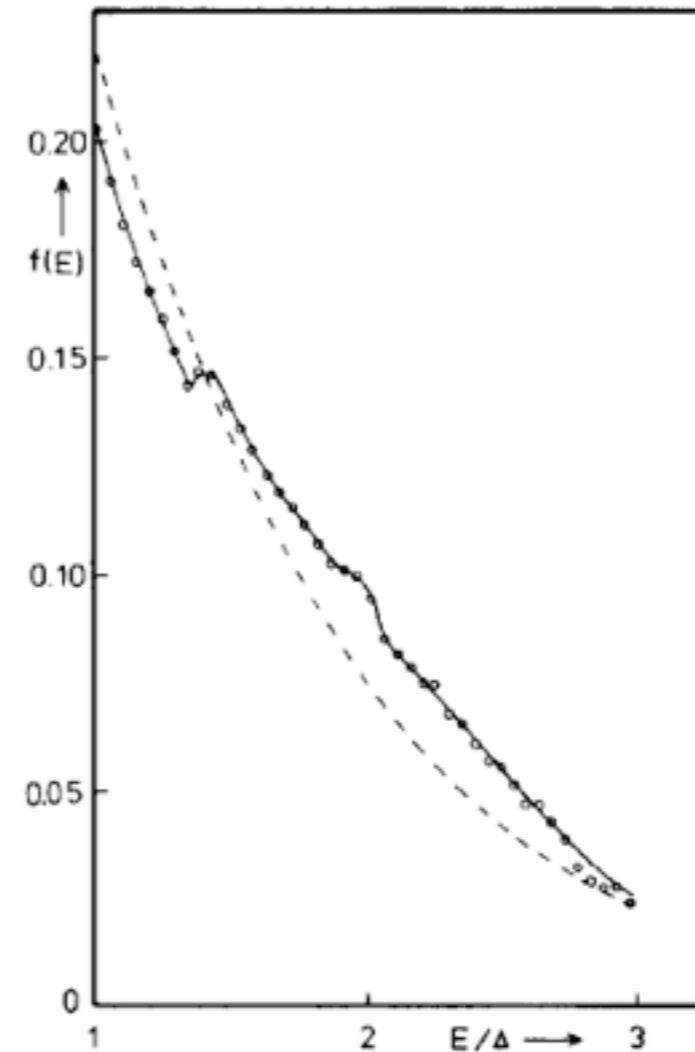
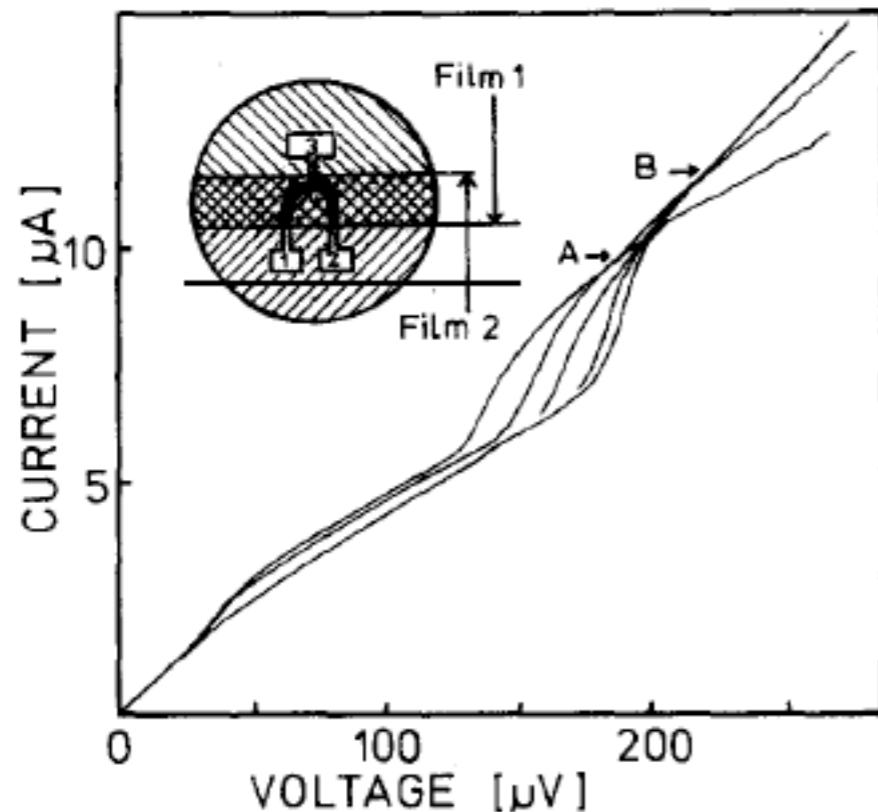
De Visser et al, arxiv 1306.4238

Full numerical analysis of Eliashberg and Chang-Scalapino



Goldie and Withington, SUST **26**, 015004(2013)

Direct measurements: horseshoe-shape tunnel-junctions

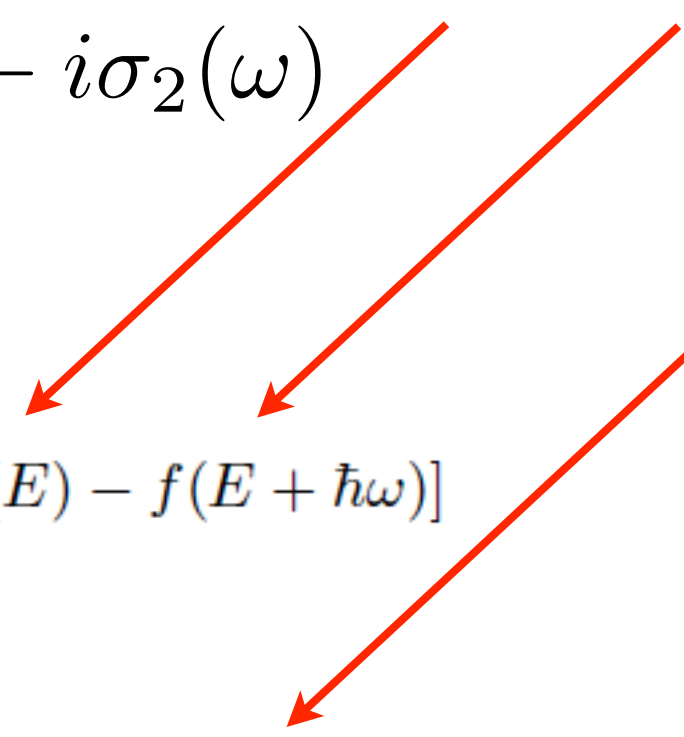


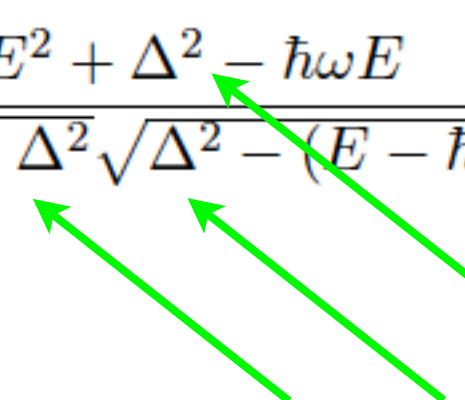
Wolter and Horstman,

Phys. Lett. **82A**, 43 (1981) and **86A**, 185 (1981)

Observables: complex impedance

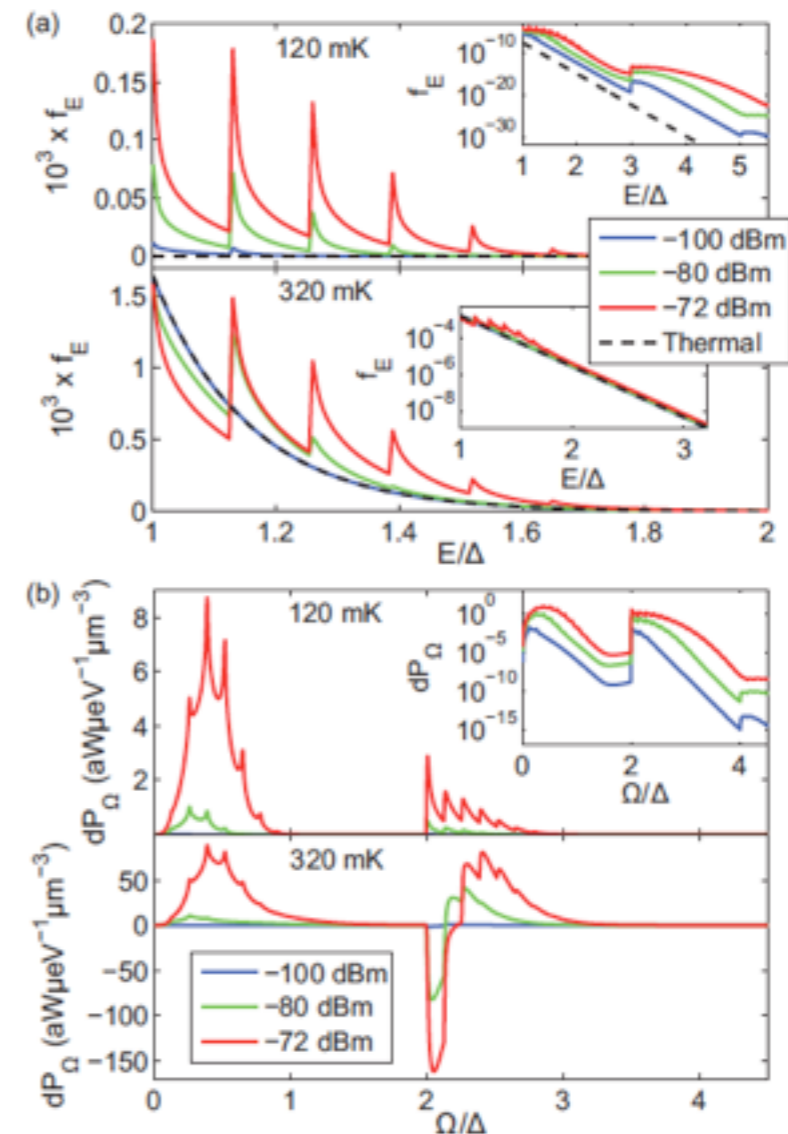
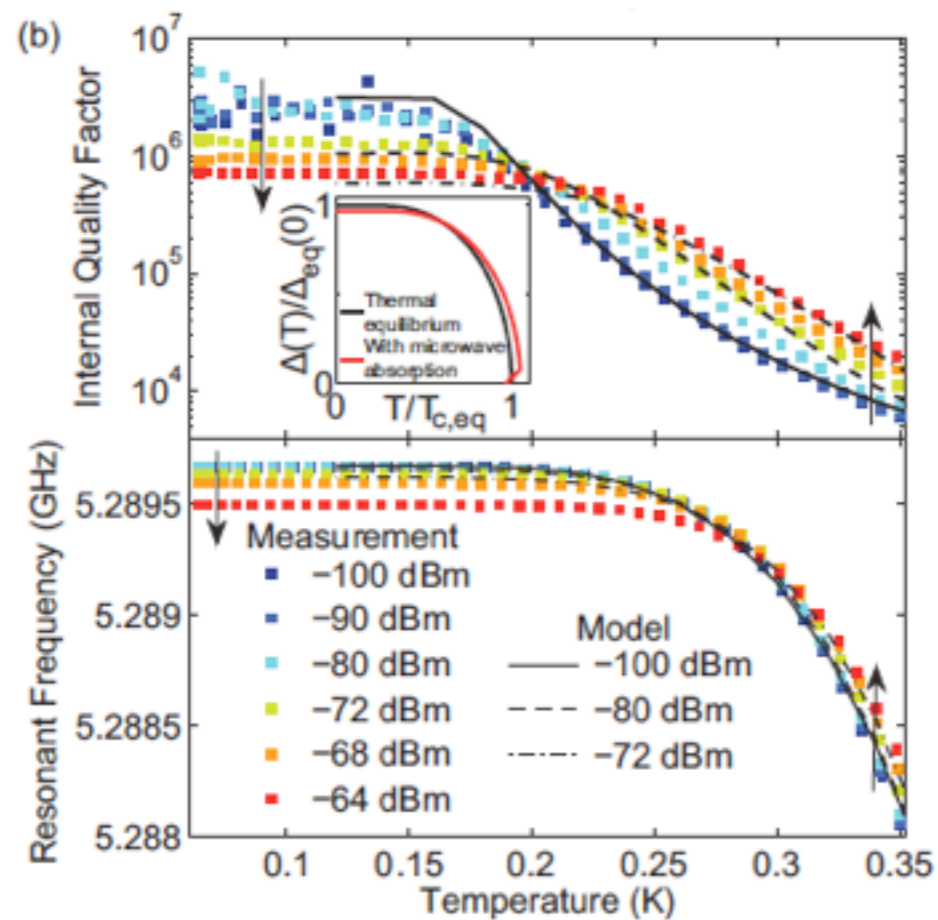
$$\vec{J}(\vec{r}) = \sigma(\omega)\vec{E}(\vec{r}) \quad \sigma(\omega) = \sigma_1(\omega) - i\sigma_2(\omega)$$

$$\frac{\sigma_1(\omega)}{\sigma_n} = \frac{2}{\hbar\omega} \int_{\Delta}^{\infty} dE \frac{E^2 + \Delta^2 + \hbar\omega E}{\sqrt{E^2 - \Delta^2} \sqrt{(E + \hbar\omega)^2 - \Delta^2}} [f(E) - f(E + \hbar\omega)]$$


$$\frac{\sigma_2(\omega)}{\sigma_n} = \frac{1}{\hbar\omega} \int_{\Delta}^{\Delta + \hbar\omega} dE \frac{E^2 + \Delta^2 - \hbar\omega E}{\sqrt{E^2 - \Delta^2} \sqrt{\Delta^2 - (E - \hbar\omega)^2}} [1 - 2f(E)]$$


$$\Delta = \Delta(f(E))$$

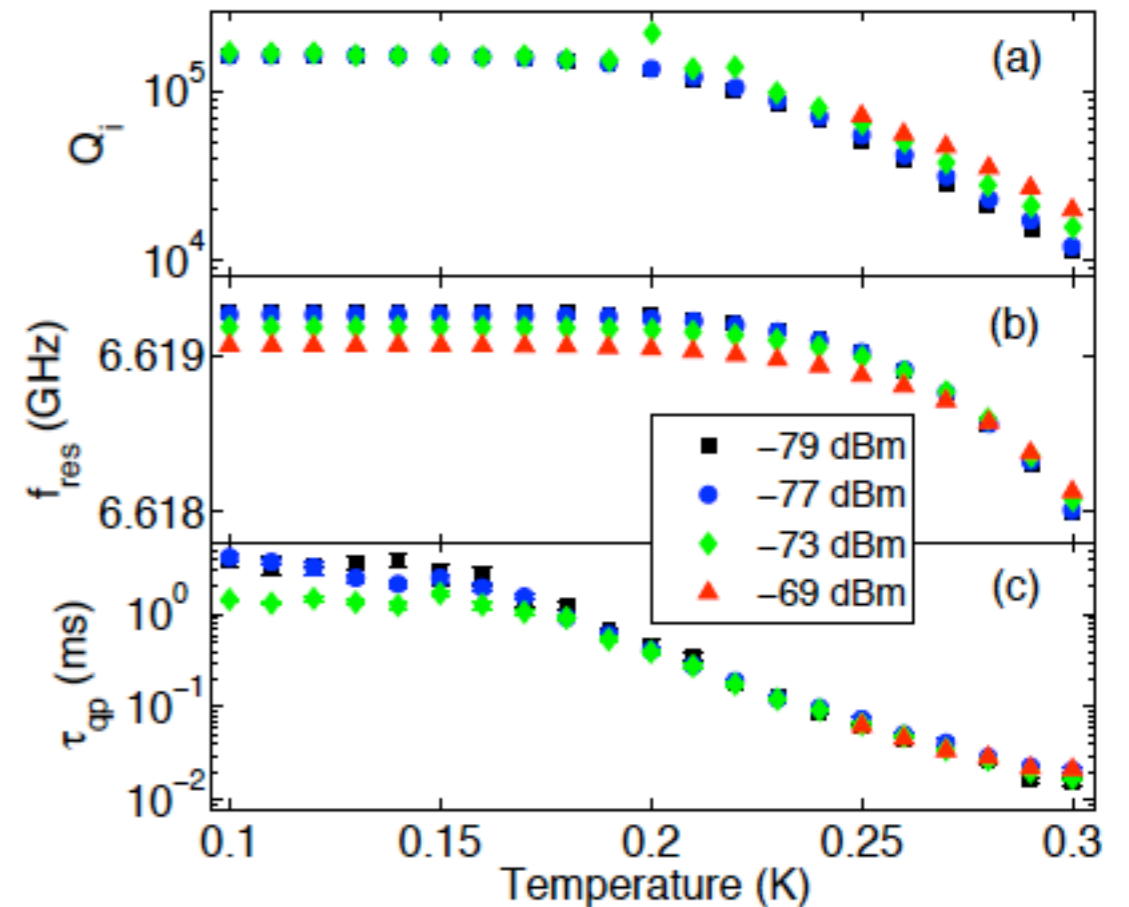
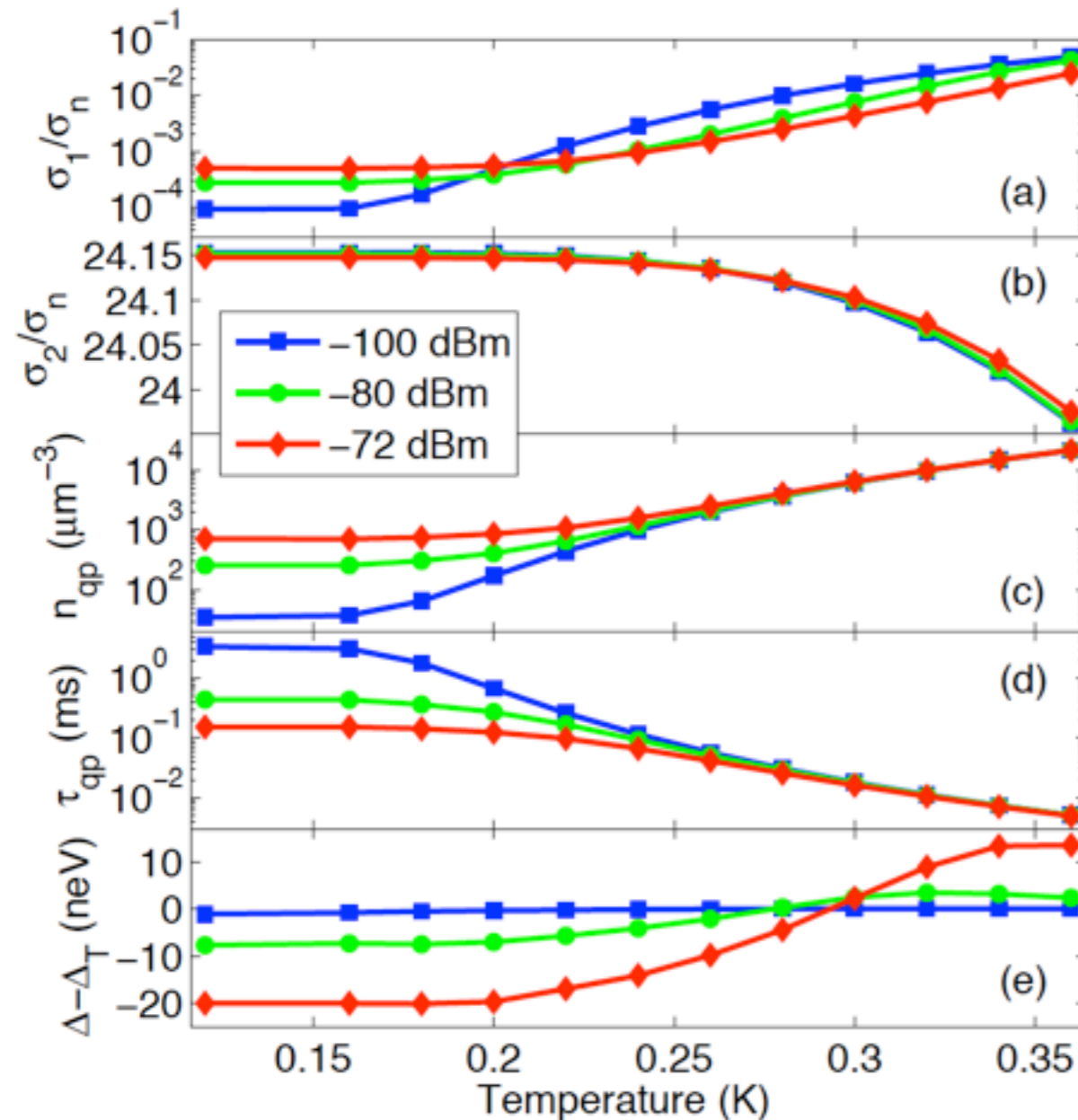
Microwave-impedance analysis for Al



Counterintuitive results:

De Visser, Goldie, Diener, Withington, Baselmans, Klapwijk, arXiv 1306.4992 and LTD-15

Response of different observables



De Visser et al, arXiv 1306.4992 and LTD-15

Any implication for TiN?

$$\vec{J}(\vec{r}) = \sigma(\omega)\vec{E}(\vec{r}) \quad \sigma(\omega) = \sigma_1(\omega) - i\sigma_2(\omega)$$

$$\frac{\sigma_1(\omega)}{\sigma_n} = \frac{2}{\hbar\omega} \int_{\Delta}^{\infty} dE \frac{E^2 + \Delta^2 + \hbar\omega E}{\sqrt{E^2 - \Delta^2} \sqrt{(E + \hbar\omega)^2 - \Delta^2}} [f(E) - f(E + \hbar\omega)]$$

$$\frac{\sigma_2(\omega)}{\sigma_n} = \frac{1}{\hbar\omega} \int_{\Delta}^{\Delta + \hbar\omega} dE \frac{E^2 + \Delta^2 - \hbar\omega E}{\sqrt{E^2 - \Delta^2} \sqrt{\Delta^2 - (E - \hbar\omega)^2}} [1 - 2f(E)]$$

$$\Delta = \Delta(\vec{r})$$

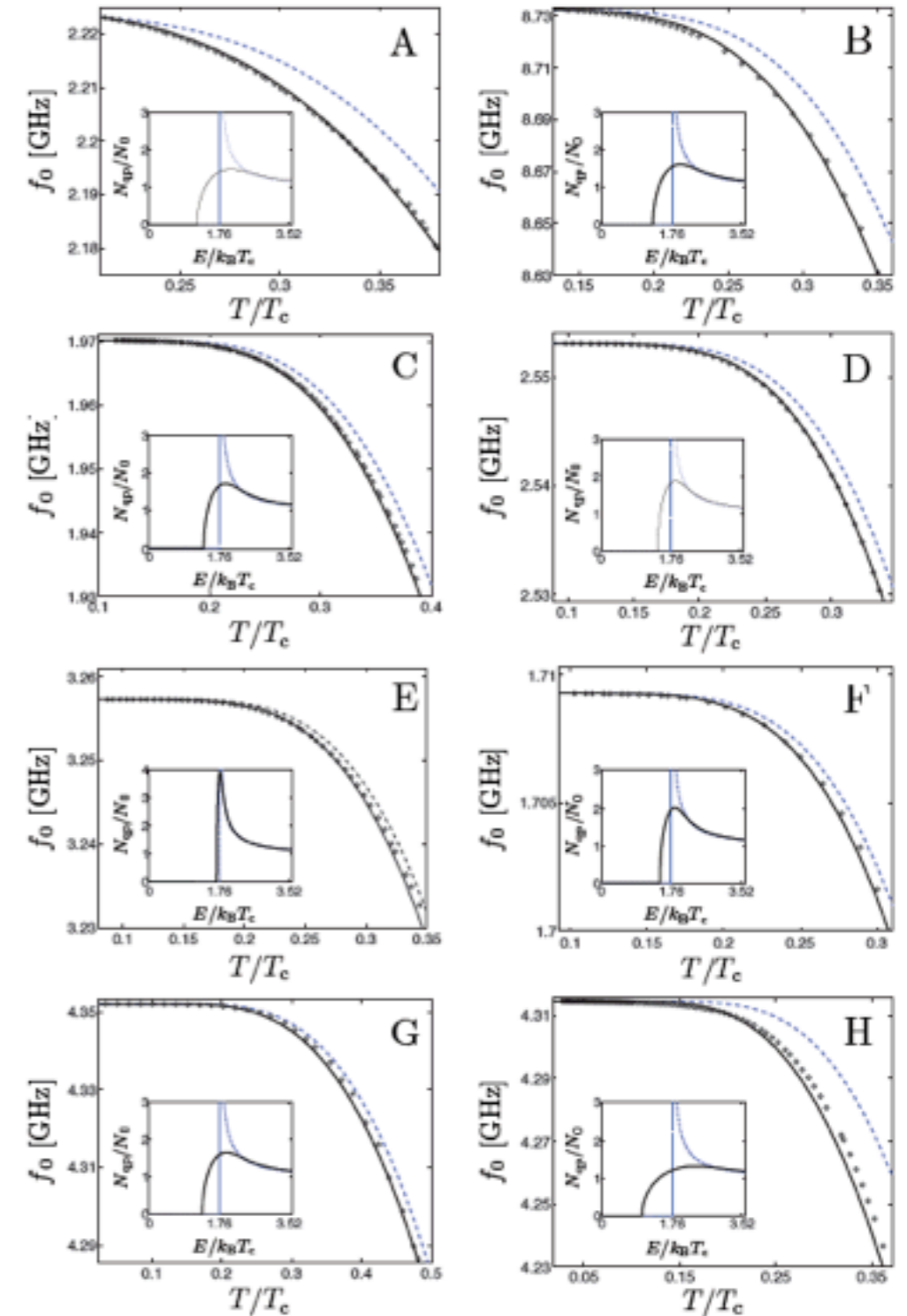
Singularities: rounded density-of-states

Temperature dependence of f_0

Driessen et al, PRL **109**, 107003 (2012)

TABLE I. Parameters of the films studied in this experiment.

Film ID	Substrate	d [nm]	b [nm]	ρ [$\mu\Omega\text{cm}$]	$k_{\text{F}}l$ [Å]	l [Å]	τ [fs]	T_c [K]	α [$k_{\text{B}}T_c'$]
<i>TiN ALD-deposited films</i>									
A	SiO ₂	6	25	380	3.3	3.4	1.5	0.22	
B	SiO ₂	11	27	356	3.5	3.5	1.2	2.2	0.17
C	SiO ₂	22	32	253	4.6	4.4	2.7	2.7	0.13
D	SiO ₂	45	37	187	6.1	5.7	1.2	3.2	0.10
E	SiO ₂	89	42	120	8.6	7.3	1.7	3.6	0.01
F	H-Si	55	44	212	6.0	6.4	1.4	3.3	0.08
<i>NbTiN sputter-deposited films</i>									
G	Sapphire	300	85	150	8.2	6.3	14.8	0.15	
H	H-Si	50	30	506	2.4	2.4	11.9	0.34	



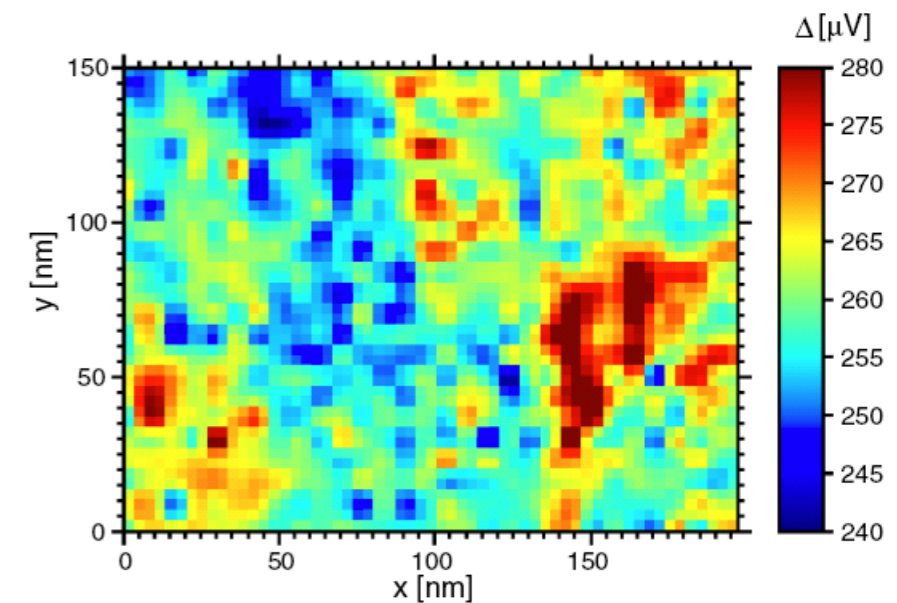
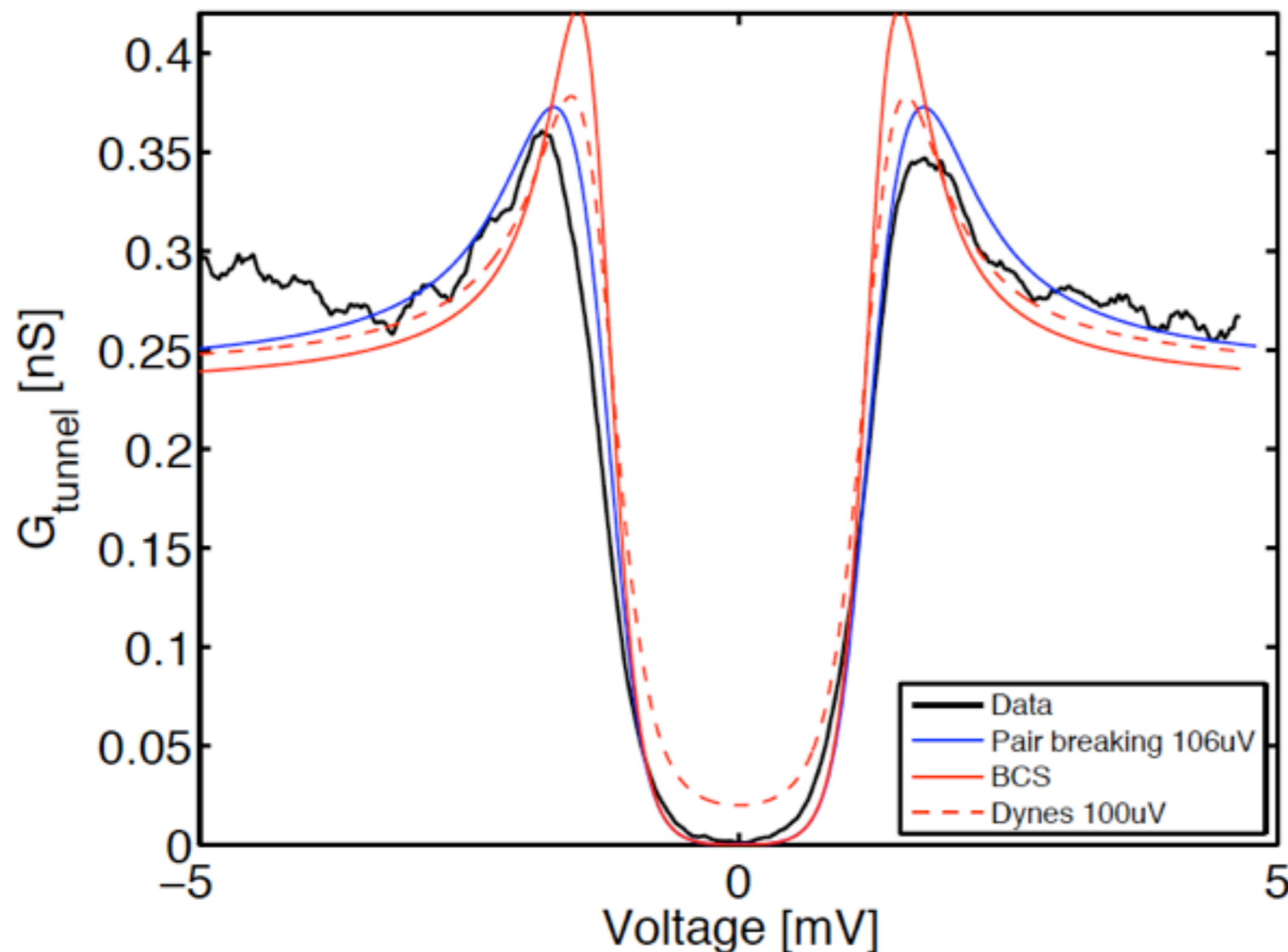
$$iE \sin \theta + \Delta \cos \theta - \alpha \sin \theta \cos \theta = 0.$$

$$\hbar\omega \frac{\sigma_2}{\sigma_n} = \int_{-\hbar\omega}^{\infty} dE g_2(E, E') [1 - 2f(E')] + \int_0^{\infty} dE g_2(E', E) [1 - 2f(E)]$$

$$\Delta = N_0 V \int_0^{k_{\text{B}}\Theta_{\text{D}}} dE \text{Im}(\sin \theta) [1 - 2f(E)]$$

Local tunneling spectra much sharper

Comparison of different models ($T_{\text{eff}} = T_{\text{thermometer}} = 1.7$)



Driessen,
unpubl. results

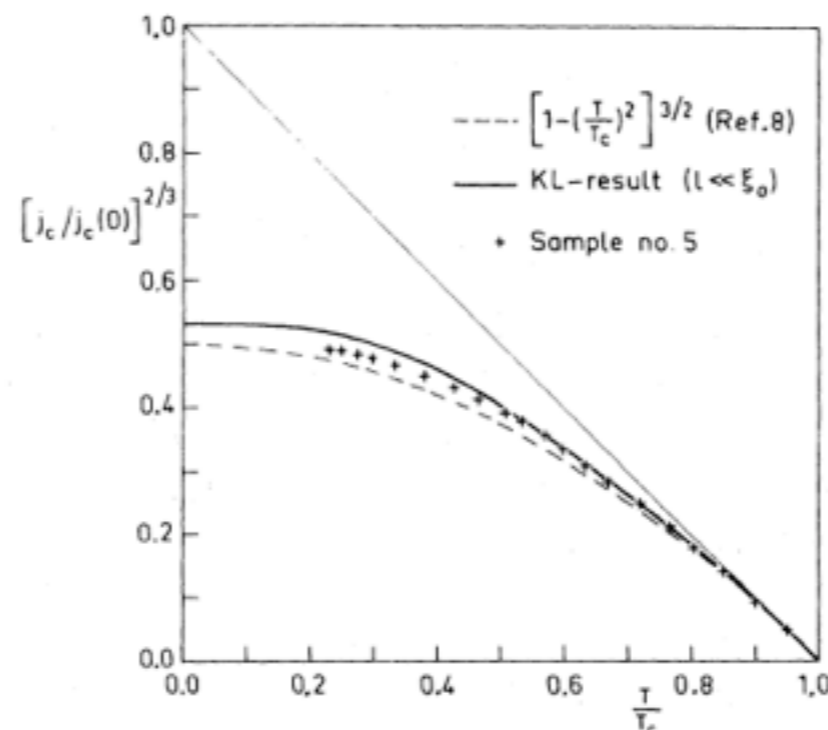
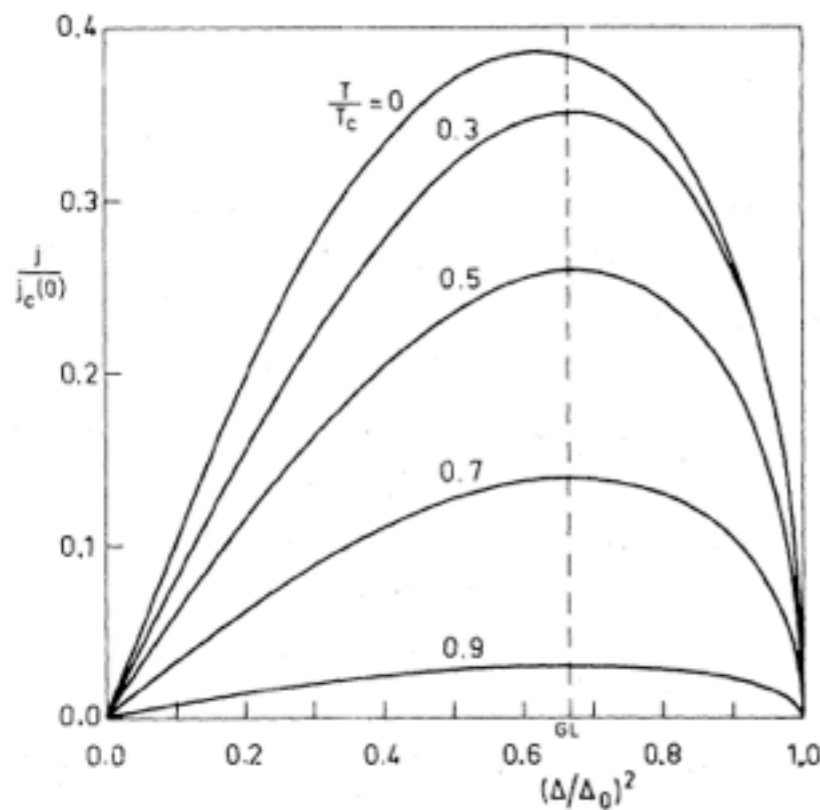
Current-carrying density-of-states

Anthore et al, PRL 2003

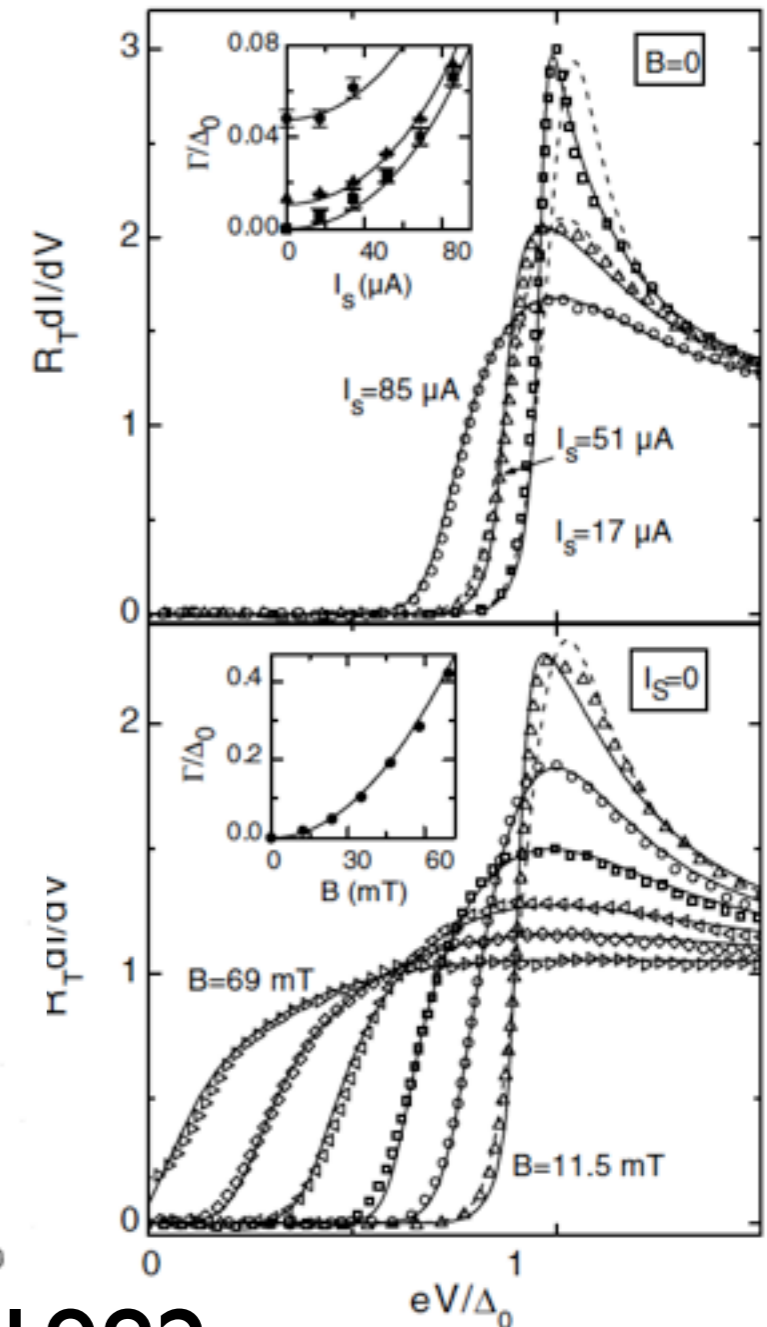
$$\frac{\hbar D}{2} \nabla^2 \theta + \left[iE - \frac{\hbar}{2D} \vec{v}_s^2 \cos \theta \right] \sin \theta + \Delta \cos \theta = 0$$

$$\vec{j}(\vec{r}) = \frac{\sigma}{eD} \int_0^\infty dE \tanh\left(\frac{\beta E}{2}\right) \text{Im}(\sin^2 \theta) \vec{v}_s$$

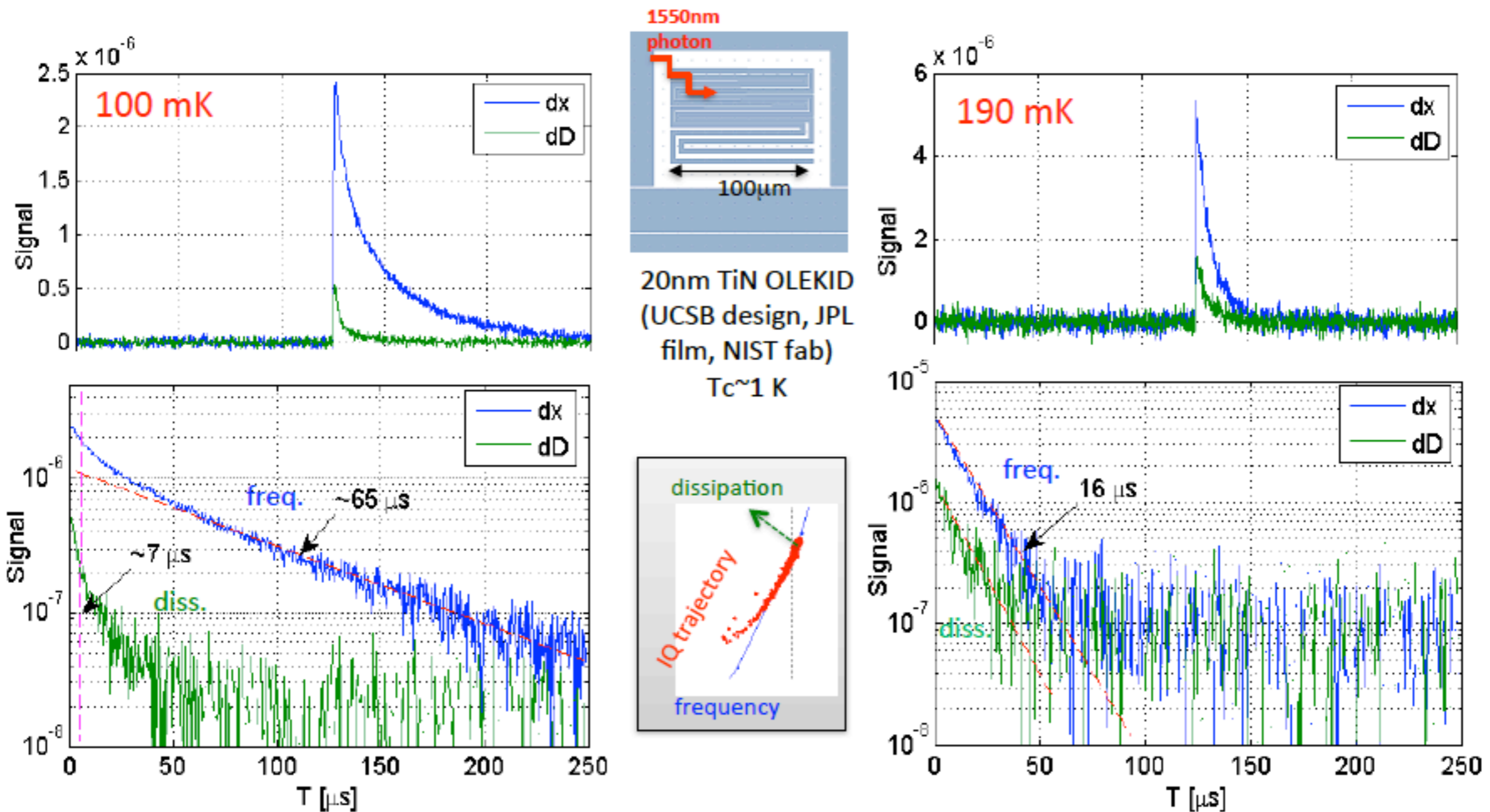
$$\Delta(\vec{r}) = N(0) V_{\text{eff}} \int_0^{\hbar \omega_D} dE \tanh\left(\frac{\beta E}{2}\right) \text{Im}(\sin \theta)$$



Romijn et al, PRB, 1982



Anomalous Response to NIR Photon



- At lower T , dissipation response shows shorter decay time than frequency response.
- At higher T , responses in the two quadratures show equal decay time.

J. Gao, et al, APL 101, 142602 (2012)

Non-equilibrium in an inhomogeneous system?

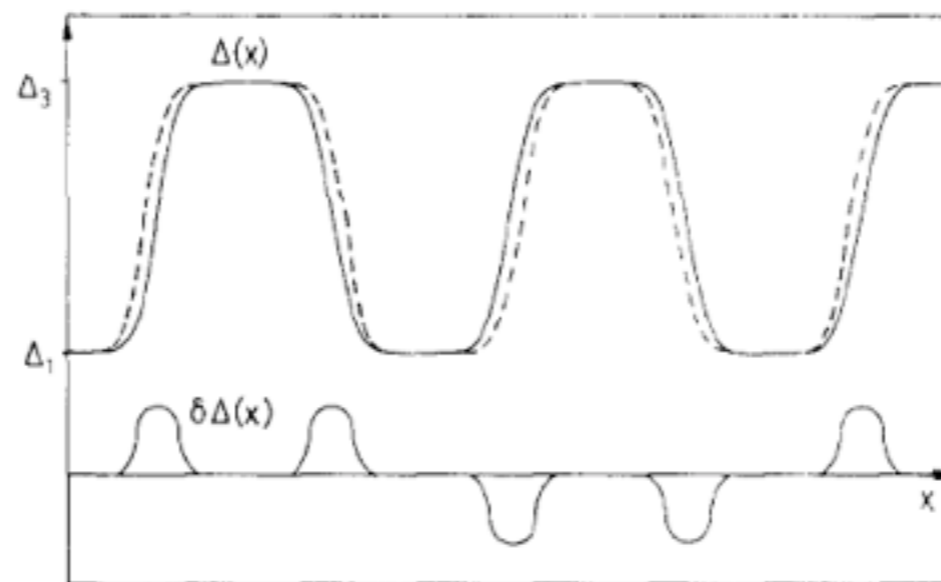


Fig. 3. A periodic structure of a conserved quantity with large wavelength may decay by coarsening [30].

G.Schön, Physica **109&110B**, 1677 (1982)

U.Eckern, A.Schmid, M.Schmutz, and G.Schön, J.

Low Temp. Phys. **36**, 643 (1979)

Summary

- Detailed understanding based on $f(E)$ is possible for Al
- Nonequilibrium analysis for Al very mature and usable.
- (The hybrids, which use NbTiN with Al equally successful)
- Robustness for TiN *vis à vis* such an analysis remains to be seen (See also Juan Bueno's poster).