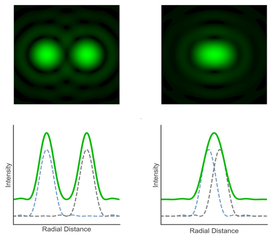
Rayleigh Criterion Applied to Astronomical Time Series

Ramirez Delgado, V.[1], Caicedo Vivas J.[1,2], Dodson-Robinson S.[1,3], Haley C.[4]
[1] Department of Physics & Astronomy, University of Delaware, Newark, DE
[2]Universidad del Valle, Cali, Valle del Cauca, Colombia
[3]Bartol Research Institute, Newark, DE
[4]Mathematics and Computer Science Division, Argonne National Laboratory, Lemont, IL

Introduction

- In optics, the **Rayleigh criterion** states that two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other (image below).
- In power spectrum analysis both the concept of resolution and the Rayleigh criterion are applicable
- Previous astronomical studies by Loumos and Deeming (1978) and Kovacs (1981) proposed a minimum separation between two frequencies of $|f_1 - f_2| \cong 1.45\Re$ ($\Re$ defined below)
- Unevenly spaced time series **can result in periodograms with significant false peaks that can be mistaken as signals arising from physical processes.**
- Using the Rayleigh criterion to check if frequencies are independent from each other can help avoid reporting spurious peak as real signals

Image Credit: Edinburgh Instruments



Mathematics

- Application of Generalized Lomb-Scargle periodogram (GSLP, Zechmeister & Kürster 2009) to RV and photometric time series.
- Definition of the Rayleigh resolution in the frequency domain in terms of the time baseline

$$\Re = \frac{1}{T} \quad (1)$$

- Rayleigh Criterion for harmonic analysis

$$|f_1 - f_2| \geq C\,\Re, \quad (2)$$

where C is a constant taken to be between 1.45 to 2 (Thomson & Emery 2014) and set equal to 2 in our analysis following our results from **Fig. 2**.

- **Corollary 1:** for an oscillation to be detected using a periodogram, it must be distinguishable from zero frequency according to the Rayleigh criterion.
- **Corollary 2:** oversampling the frequency grid does not increase the Rayleigh resolution.

Results

- We apply the Rayleigh criterion and our corollaries mentioned above to three toy models with known frequencies of oscillation.
- Lastly, we explore the Rayleigh criterion with the archival data of Barnard's star radial velocities (RV) and the Kepler light curve of active star KIC 891916.
- We determine that checking if our frequencies are separated by less than $2\Re$ avoids false detections of spurious signals

Synthetic Time Series

Splitting of Periodogram Peak:

- Create periodic signal $y(t)$ designed to mimic an RV time series.
- Begin with a realization of 500 samples and a uniform time separation. We omit 420 points from the sample, while preserving the first and last data point, to obtain $y'(t)$ with the same Rayleigh resolution.
- The thinned series' periodogram has a close neighboring peak to the true frequency, having the second highest power.
- This new peak is spurious and statistically indistinguishable from the original one since it fails the Rayleigh criterion.
- **Spurious splitting of a single peak into doublets can happen when there is a large variation in Δt**, as is the case in this example.

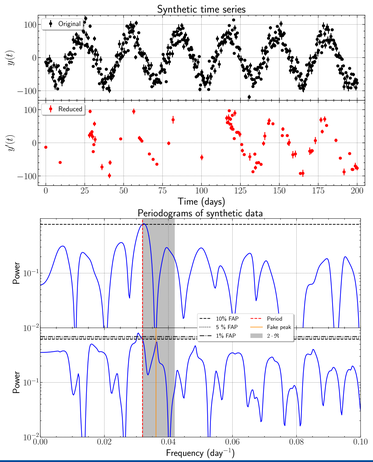
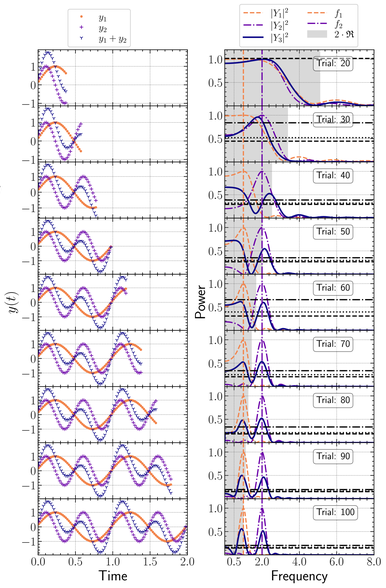


Fig. 1

Varying time length:

- Observe the performance of the GLSP at different values of T.
- Start with a set of 100 timestamps of uniform $\Delta t = 2$. We then create 100 new sets of timestamps by adding a new data point in each trial keeping Δt unchanged.
- Have three signals, two independent sinusoids (y_1, y_2) and the sum of the two $(y_3 = y_1 + y_2)$.
- **Once $f_2 = 2\Re$ the periodogram peak aligns with the correct frequency.**
- Astronomical time series may contain more than two periodic components, and other factors that can diminish the efficiency of the periodogram.

Fig. 2 (scan QR code to see animation)



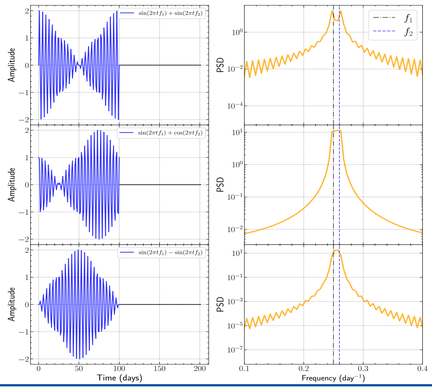
Oversampling the frequency grid:

- Oversampling refers to creating a frequency grid with $\Delta f < \Re$, equivalent to zero-padding in the even sampling case.
- We demonstrate the veracity of Corollary 2 by creating a time series of 100 observations with 2 sinusoids

$$y(t) = \sin(2\pi\, t\, f_1) + \cos(2\pi\, t\, f_2 + \phi)$$

where $|f_1 - f_2| = \Re$ and zero-padding the time series (**Fig.3**).
- We evaluate our time series for three different values of $\phi = [0, \frac{\pi}{2}, \pi]$ and compute the periodogram for each case.
- Our results show two of the three cases do not resolve the individual peaks, showing that zero-padding is not guaranteed to improve periodogram resolution.

Fig. 3



Archival Time series

Barnard's Star:

- Old M-dwarf star with a reported planet detection with orbital period of 232.8 ± 0.4 days (Ribas et al. 2018).
- The long-term activity cycle has a reported period of 6600 days, a separation of $1.13\Re$ from zero frequency.
- Used a cubic polynomial to subtract the long-term activity cycle from the RV observations.
- Residuals have smaller standard deviations and more suppression of low-frequency peaks than residuals from the sine model.
- Our polynomial fit shows that observers **cannot assume that a period is correctly measured for signals with $f < 2\Re$**, or even that such signals are periodic

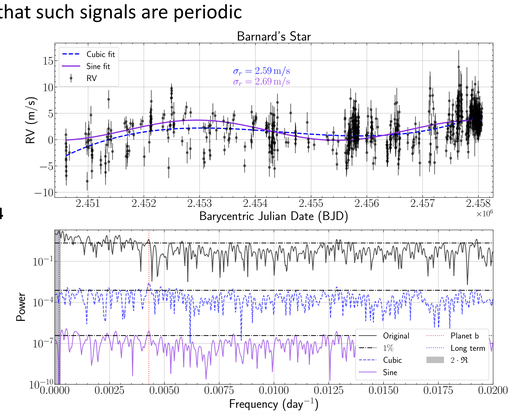


Fig. 4

KIC 891916:

- Part of the sample of 40,661 active stars studied in Reinhold et al. (2013) observed by the Kepler mission and used to study differential rotation.
- Reinhold et al. (2013) computed the GLSP of light curves binned in two-hour windows and subtracted a sinusoid at the frequency with the highest power. This procedure was repeated four times.
- Of the five periods recorded, the first period P_1 is explained as surface rotation, and if another period is within $\pm 30\%$ P_1 it is attributed to differential rotation.
- The reported **differential rotation period is less than $2\Re$ from P_1** and the peak is not statistically significant.

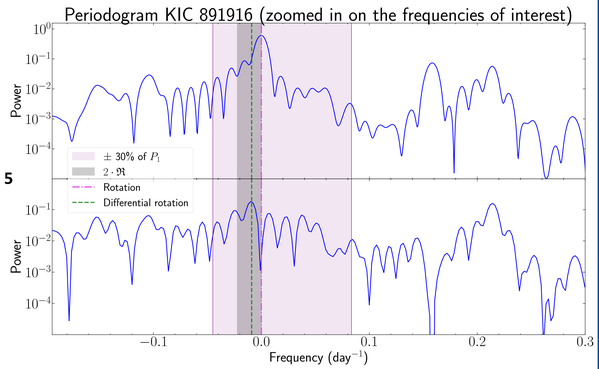


Fig. 5

Conclusions

- The Rayleigh Criterion is a fundamental part of harmonic analysis and should be more applied in the analysis of astronomical time series.
- Our results highlight that closely spaced frequencies may appear as a single periodogram peak when the time baseline is not long enough to resolve them.
- Closely spaced peaks might not be independent from each other when dealing with unevenly spaced data sets.
- Using $C = 2$ in equation (2) guarantees that a signal is observed for two full cycles but as seen in the literature in some cases a frequency separation of $1.45\Re$ is enough to resolve two independent frequencies.
- Applying the Rayleigh criterion can avoid false positive detections of planets in RV time series when there is not enough time coverage of a planet's orbit.

Acknowledgments & References

This work is supported by the University of Delaware Graduate College through the Unidel Distinguished Graduate Scholar Award.

References:

Kovacs G., 1981, Ap&SS, 78, 175
Loumos G. L., Deeming T. J., 1978, Ap&SS, 56, 285
Rayleigh L., 1879, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 8, 261
Reinhold T., Reiners A., Basri G., 2013, Astronomy & Astrophysics, 560, A4
Ribas I., et al., 2018, Nature, 563, 365