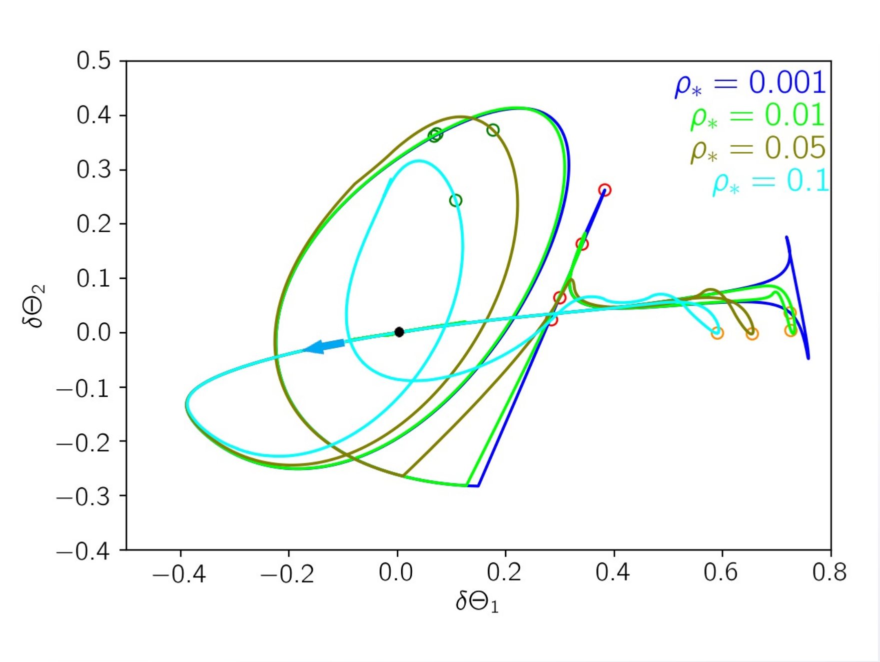
Astrometric Microlensing with *Roman*:

a Public Code and Perspectives for Detection



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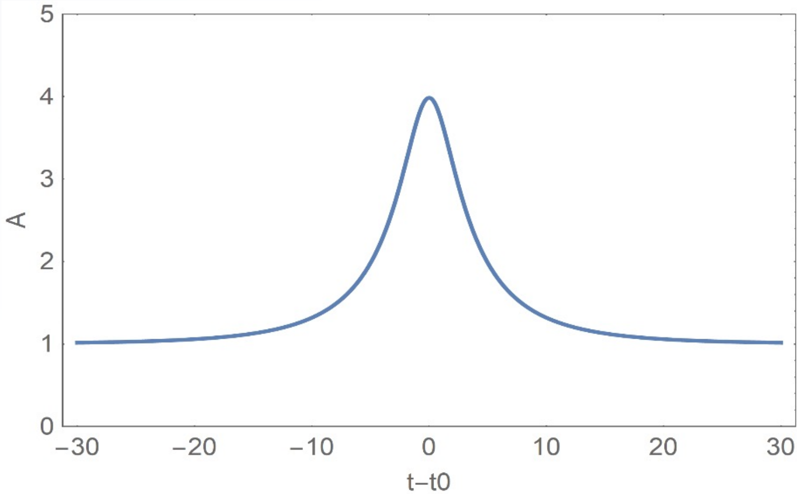
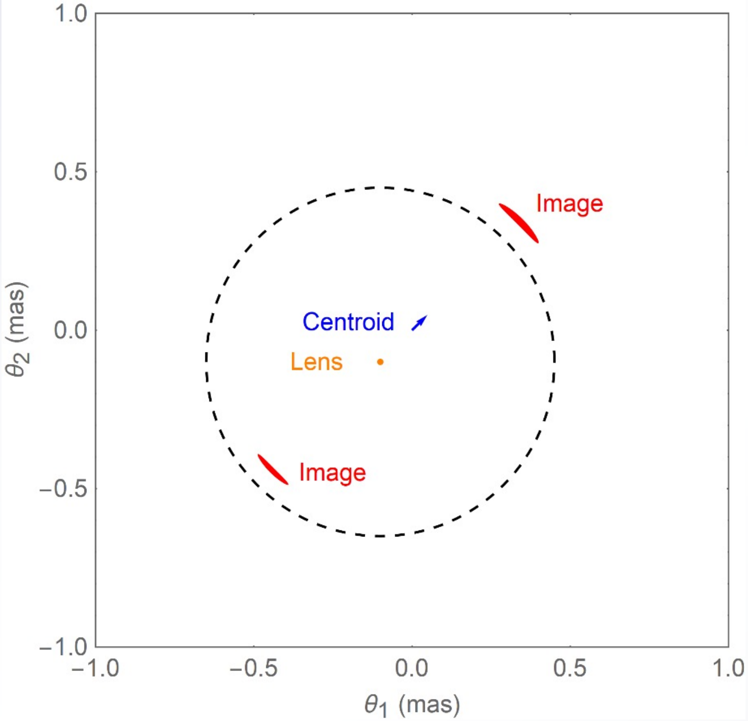
Astrometric Microlensing

- Photometric Microlensing measures the amplification of the **flux** of a background source
- Individual images remain **unresolved** (order 1 mas needed!)

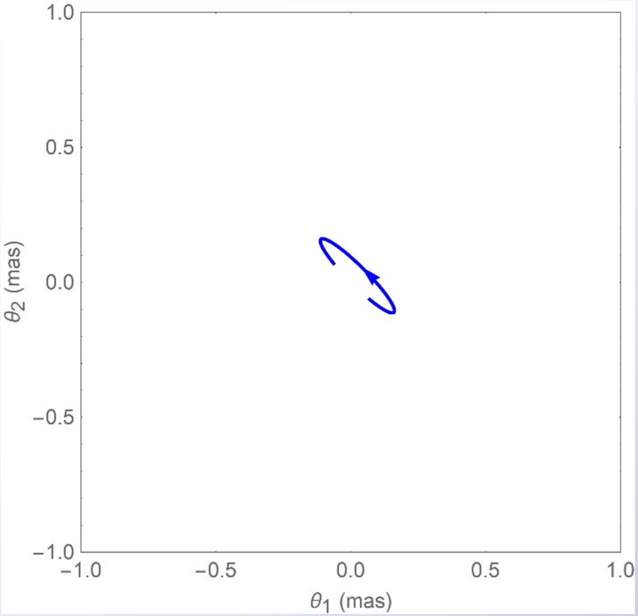




- Precision astrometry may detect the **centroid shift**!

$$\boldsymbol{\delta\theta} = \frac{\boldsymbol{u}}{u^2+2}\theta_E$$

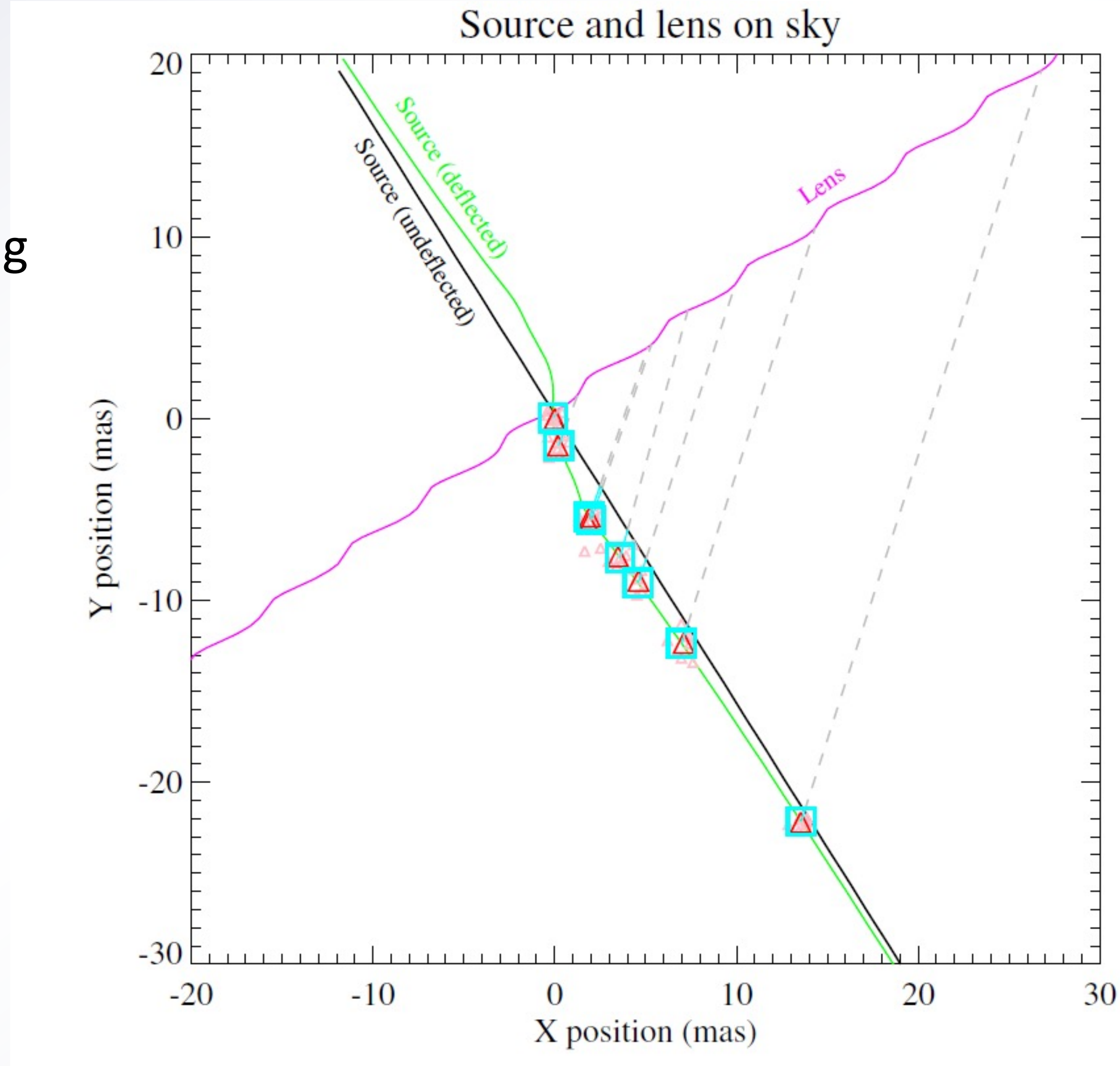
$$\delta\theta_{max} = \frac{\theta_E}{2\sqrt{2}} = 0.35\theta_E$$



- **Direct measurement of the Einstein angle $\boldsymbol{\theta_E}$!**

MOA-2011-BLG-191/OGLE-2011-BLG-0462

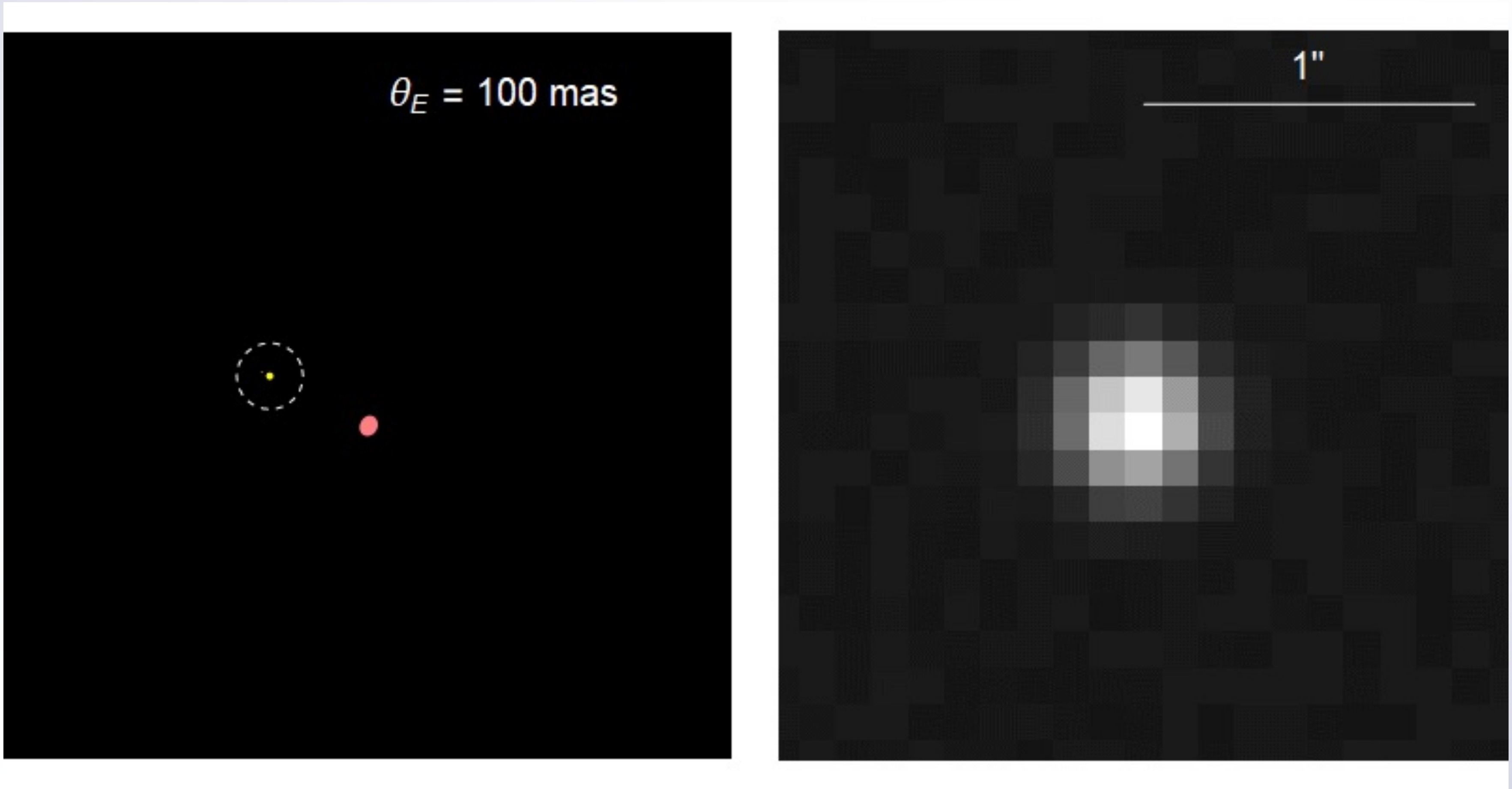
- Astrometric microlensing detected by HST.
- An isolated BH of 7 $M_\odot$



- *Sahu et al. arXiv:2201.13296, Lam et al. arXiv:2202.01903*

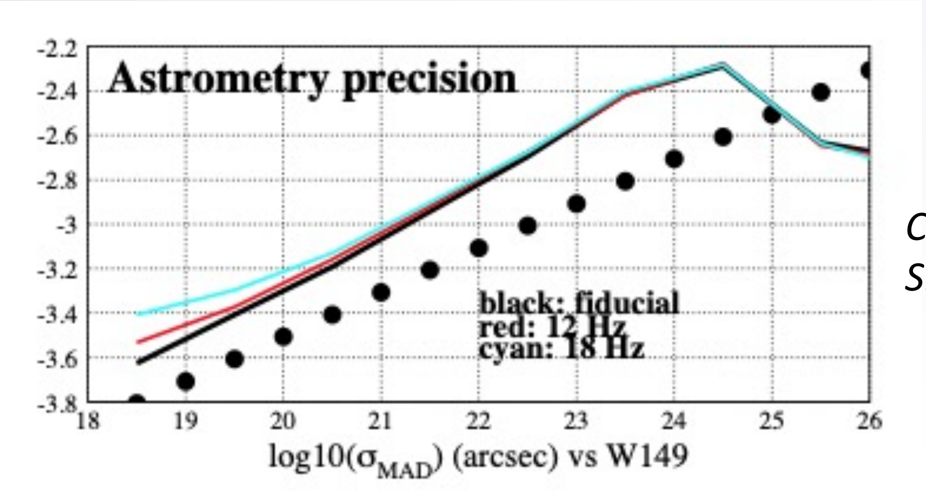
Astrometric Microlensing with *Roman*

- Simulation of a microlensing event with an (exaggerate) $\theta_E = 100$mas on *Roman* pixel size and PSF for visualization purposes.

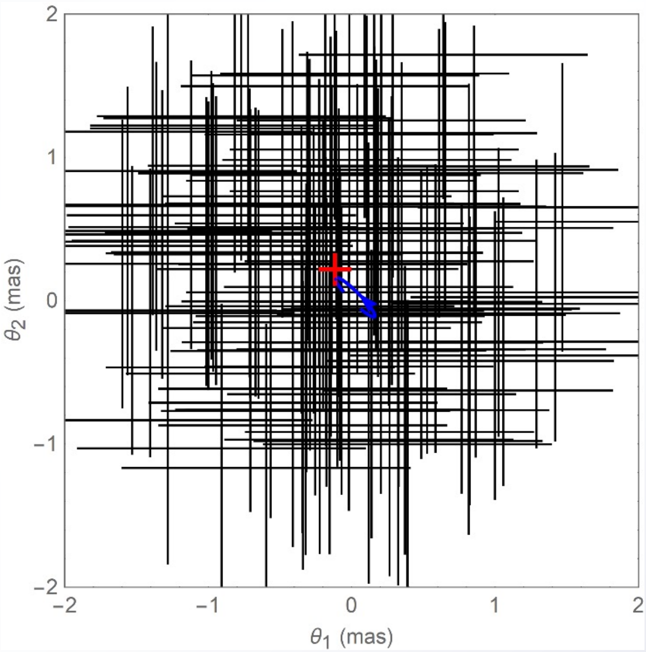


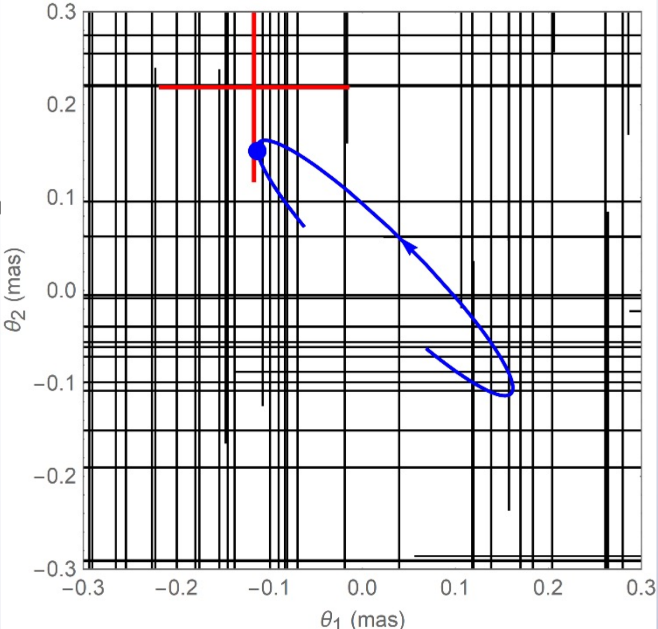
- *Roman* should be able to reach an **astrometric precision of 1 mas**.

Astrometric Microlensing with *Roman*



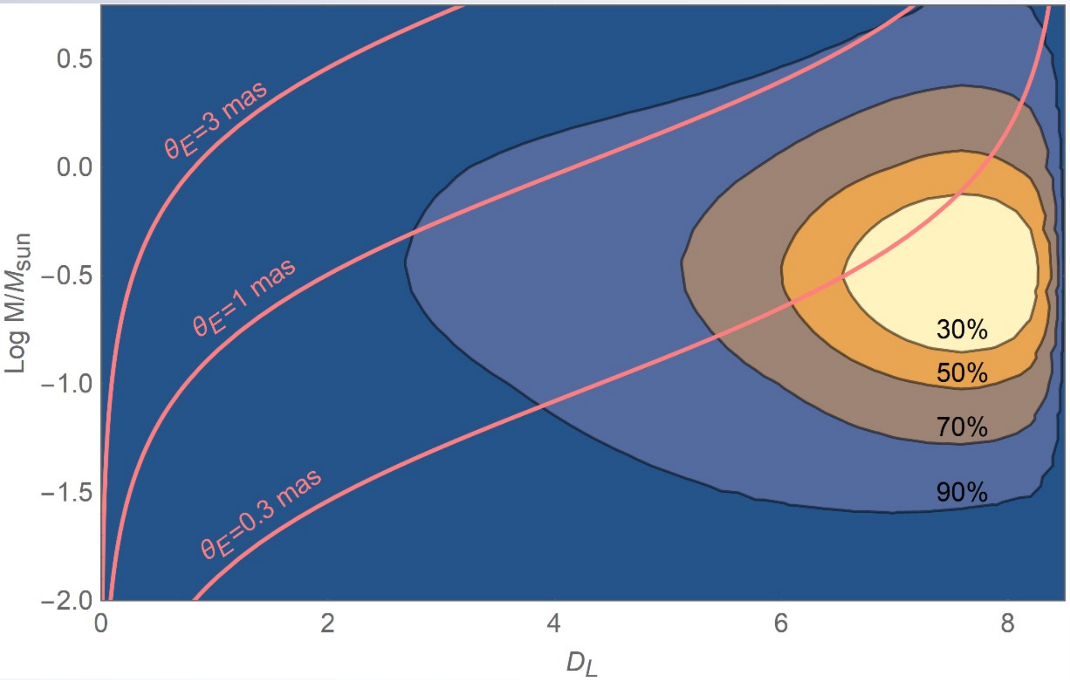
Credit to
S. Calchi Novati





- *Roman* should be able to reach an **astrometric precision of 1 mas** on *W149=21* mag stars.
- With a 15 min sampling, we have nearly **100 images per day**.
- Averaging on 100 measurements, the uncertainty is reduced by a factor 10.
- However, a good understanding of all **systematics** is necessary.
- **Blending** by unresolved sources or the lens itself reduces the signal.

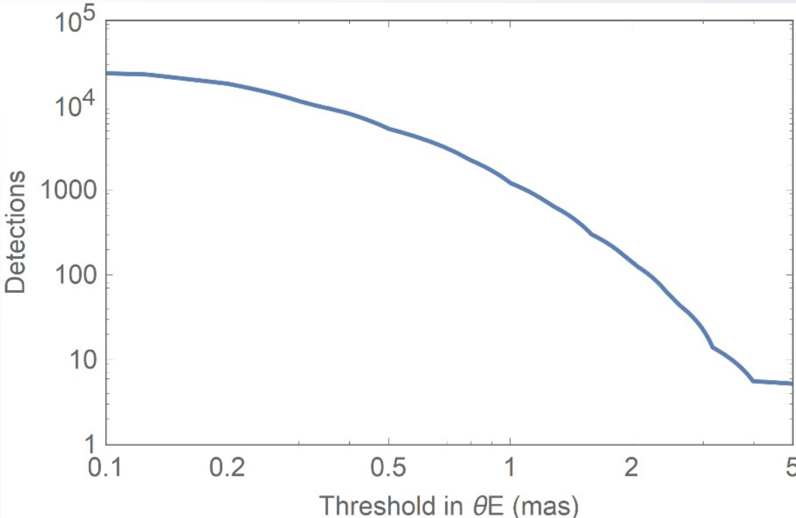
Astrometric Microlensing with *Roman*



- Distribution of microlensing events rate in the plane (D_L, M).
- Detection thresholds for θ_E. [Galactic model from Dominik (2006), fiducial source at $D_S = 8.5$ kpc]

- The **number of detections** quickly drops with the threshold:

Thr. for θ_E	Detections
0.3 mas	11884
1 mas	1212
3 mas	31



Information in Astrometry

- **Astrometry** may yield θ_E for nearby massive lenses
- Complementary to **finite source** effect $\rho_* = \theta_*/\theta_E$.
- Optimal regime for **annual parallax** π_E → Mass measurement!

$$M_L = \frac{c^2 \mathrm{au}\, \theta_E}{4G\, \pi_E}$$

- Good synergy with lens flux measurement technique for luminous lenses.
- **Strategic method for dark lenses** (black holes or other stellar remnants).

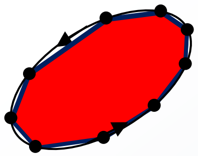
- **Simultaneous modeling** of photometry and astrometry required.
- Additional effects must be taken into account
  - Finite source effect
  - Limb darkening
  - Binary or planetary lenses
    - Parallax, orbital motion, xallarap

Computation in Microlensing

- VBBinaryLensing 3.0 *(VB 2010; VB et al. 2018; VB et al. 2021)*
  - Microlensing computation including advanced methods
  - Public code (https://github.com/valboz/VBBinaryLensing)
  - Written in C++, importable in Python

- VBBL 3.0 at the core of several modeling platforms:
  - RTModel (http://www.fisica.unisa.it/GravitationAstrophysics/RTModel.htm)
  - pyLIMA (https://github.com/ebachelet/pyLIMA, Bachelet et al. 2018)
  - MulensModel (https://github.com/rpoleski/MulensModel, Poleski & Yee 2019)
  - muLAn (https://github.com/muLAn-project/muLAn, Cassan & Ranc 2017)
  - Adopted as the base computation code in *Roman* microlensing pipeline

VBBinaryLensing 3.0

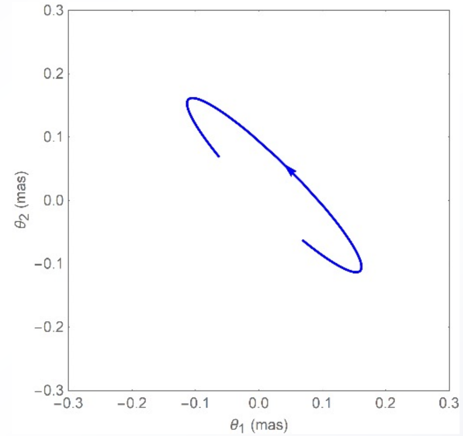


- **Lens inversion**:
  inversion of lens equation by Skowron-Gould algorithm.
- **Contour integration**:
  area of the images from Green's theorem on the boundaries.
- **Parabolic correction**:
  Residuals are of order Δx^5 instead of Δx^3.
- **Error control**:
  Error estimators for each boundary arc to assess the accuracy.
- **Optimal sampling**:
  Sampling is increased where error is largest.
- **Limb darkening**:
  Computation repeated on concentric annuli with optimized radii.
- **Decision tree**:
  Contour integration triggered when quadrupole correction high.
- **Higher orders**:
  Parallax, satellites, orbital motion, xallarap available.

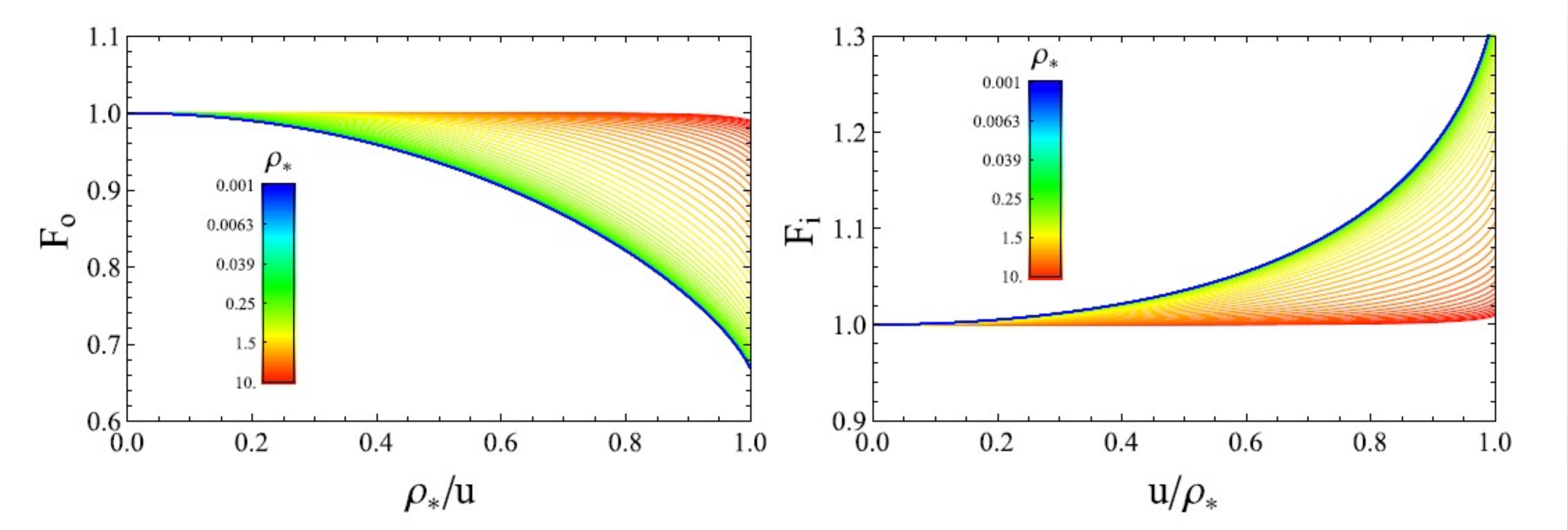
Single-lens + Finite-source

- The centroid for a single-lens describes an ellipse
- The finite-source effect can be taken into account through pre-calculated tables of elliptic integrals.

Much faster!

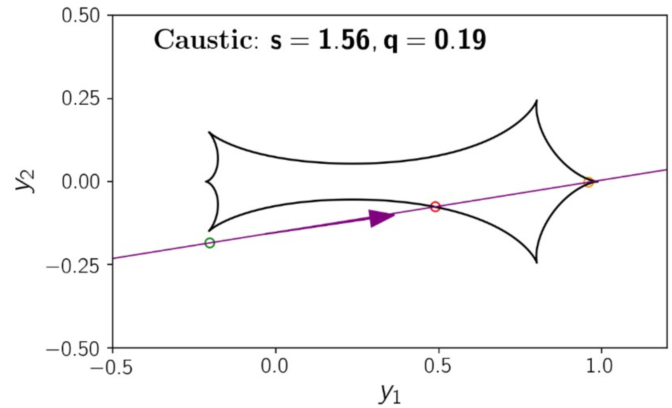


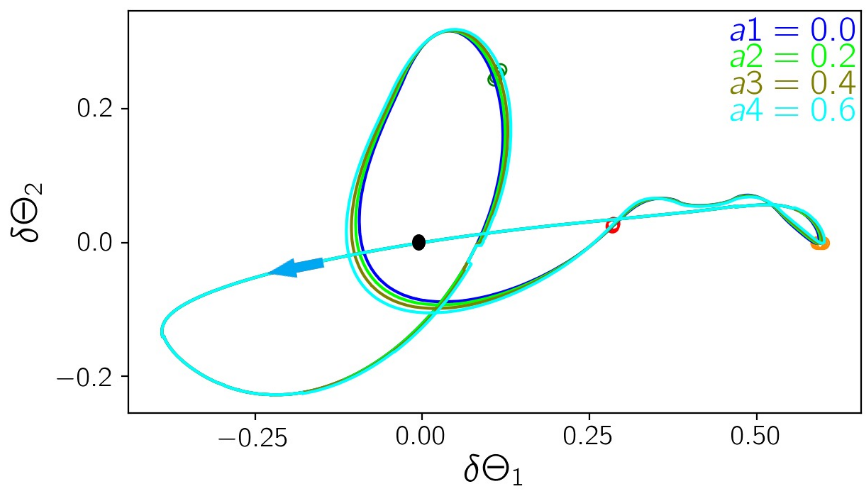
$$\Theta(u,\rho_*) = \begin{cases} \frac{u(u^2+3)}{u^2+2} F_o(\rho_*/u, \rho_*) & u > \rho_* \\ \left(1 - \frac{1}{4+\rho_*^2}\right) u F_i(u\rho_*, \rho_*) & u < \rho_* \end{cases}$$

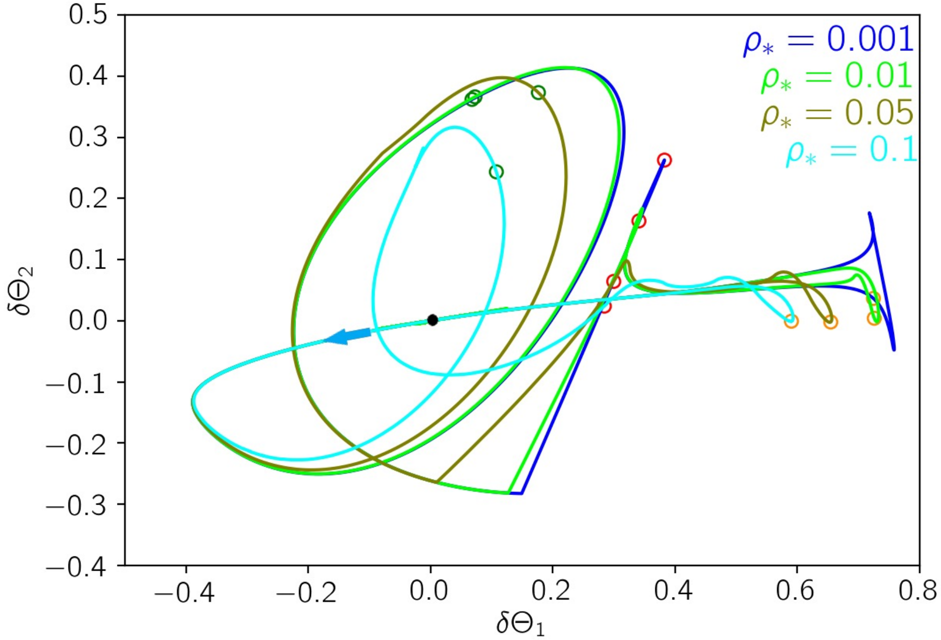


Binary-lens + Finite-source

- For binary lenses, the creation of **additional images** brings to **sudden jumps**.
- Finite-source effect washes out these jumps.
- **Limb darkening** leads to small adjustments for larger sources.







Example of use

```
#include <stdio.h>
#include “VBBinaryLensingLibrary.h”

int main()
{
   VBBinaryLensing VBBL;
   double Mag,s,q,y1,y2,rho,a1,accuracy;
   s=0.8; //separation
   q=0.1; // mass ratio
   y1=0.01; // source position
   y2=0.3;
   rho=0.01; // source radius

   VBBL.a1=0.51; // Linear limb darkening coefficient
   VBBL.Tol=1.e-2; // Accuracy goal
   VBBL.astrometry = true;
   Mag=VBBL.BinaryMag2(s,q,y1,y2,rho);

   printf("Magnification = %lf\n",Mag);
   printf(“Centroid = (%lf,%lf)\n",VBBL.astrox1,VBBL.astrox2);
   return 0;
}
```

Conclusions

- The potential to detect **astrometric microlensing** by the *Roman* Galactic Exoplanet Survey is probably high, but yet to be fully investigated.
- Depending on the accuracy threshold and our control of systematics, we may have from **hundreds to thousands detections**.
- **Mass measurement** for hundreds of dark lenses (black holes, remnants, brown dwarfs, …)
- **VBBinaryLensing** has become the reference code for photometric and astrometric microlensing.

- https://github.com/valboz/VBBinaryLensing

- VB, MNRAS 408 (2010) 2188
- VB, E. Bachelet, F. Bartolic, T.M. Heintz, A.R. Hoag, M. Hundertmark, MNRAS 479 (2018) 5157
- VB, E. Khalouei and E. Bachelet, MNRAS 505 (2021) 126